A version of the quasiparticle–phonon nuclear model for even–even deformed nuclei

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The basic assumptions of the quasiparticle–phonon nuclear model are formulated and its mathematical formalism presented. The Hamiltonian, which contains finite-rank separable isoscalar and isovector multipole, spin–multipole, and tensor particle–hole and also particle–particle interactions, is transformed to a form with quasiparticle, phonon, and quasiparticle–phonon interactions. A general RPA equation is obtained, and special cases are discussed. The very complicated interactions do not make it difficult to describe the fragmentation of the one-phonon states. It is shown that three-phonon terms, added to the one- and two-phonon terms of the wave function, lead to an additional small shift of the two-phonon poles in the secular equation. The influence of a density-dependent separable interaction on the vibrational states is weak. A general description of the collective, weakly collective, and two-quasiparticle states in even–even strongly deformed nuclei is obtained.

1. INTRODUCTION

The energies and wave functions of two-quasiparticle and one-phonon states in even–even deformed nuclei were calculated during 1960–1975. A fairly good description of the currently available experimental data was obtained,[1] and predictions that were subsequently experimentally confirmed in many cases were made. It appears to us that new calculations of vibrational states in deformed nuclei are needed. We say this because of the large number of new experimental data in addition to those that include the first quadrupole and octupole states. There are experimental data on hexadecapole states, and also high-lying collective and weakly collective states. It is to be expected that many experimental data will be obtained with the new generation of accelerators, and the results of calculations may be helpful subsequently.

Vibrational states must be calculated on a new basis such as the quasiparticle–phonon nuclear model (QPNM).[2,3] The QPNM is used for the microscopic description of low-spin vibrational states with small amplitudes in spherical nuclei not far from closed shells and in strongly deformed nuclei.

Let us consider specific features of deformed nuclei. On the transition from spherical to axial symmetry, the spherical subshells are split into doubly degenerate single-particle states. This splitting of the subshells leads to a decrease in the matrix elements of some operators between one-particle wave functions of the axisymmetric Woods–Saxon potential compared with the matrix elements of the same operators between the wave functions of the spherically symmetric Woods–Saxon potential. Such a decrease in the matrix elements has a strong effect on the vibrational states of the deformed nucleus.

The wave functions of the excited states of the deformed nuclei have the form

$$\Psi^I_{MK}(\nu)=\sqrt{\frac{2I+1}{16\pi^2}}\{D^I_{MK}\Psi_\nu(K^\pi\sigma=+)+(-1)^{I+K}D^I_{M-K}\Psi_\nu(K^\pi\sigma=-)\}.$$

We shall restrict our investigations to an intrinsic wave function $\Psi_\nu(K^\pi\sigma)$ with good quantum number K, parity π, and $\sigma=\pm1$. It is possible to take into account Coriolis interaction when necessary.

A specific property of deformed nuclei is that one-phonon states with the same K^π can be formed as the result of different multipole and spin-multipole interactions. One-phonon states of electric type or states of natural parity with fixed K^π can be described by multipole interactions with $\lambda\mu=KK$, $K+2K$, $K+4K$, etc., and by spin-multipole interactions with $\lambda\lambda_\mu=KKK$, $K+2K+2K$, etc. One-phonon states of magnetic type or states of unnatural parity can be described by the spin-multipole interaction $\lambda' LK$ with $\lambda'=L\pm1$ and by a tensor interaction. If in deformed nuclei, as in spherical nuclei, independent phonons of electric and magnetic type are introduced, then the number of states is doubled. To avoid doubling a phonon operator that combines the electric and magnetic parts was introduced in Ref. 4, 5.

In this review we present the mathematical formalism of the QPNM for the microscopic description of even–even strongly deformed nuclei. The main assumptions of the model and the Hamiltonian are formulated in Sec. 2. The general RPA equation and some special cases are given in Sec. 3. In Sec. 4 we introduce the wave function of nonrotational excited states and obtain the basic equations of the QPNM.

2. BASIC ASSUMPTIONS IN THE QPNM

The initial QPNM Hamiltonian for nonrotational states of deformed nuclei consists of the average field of the neutron and proton systems in the form of the axisymmetric Woods–Saxon potential, monopole pairing, and isoscalar and isovector particle–hole (ph) and also particle–particle (pp) multipole, spin-multipole, and tensor interactions between the quasiparticles. The Hamiltonian can be written in the form

$$H=H_{\rm s.p.}+H_{\rm pair}+H_M+H_S+H_T. \tag{2.1}$$

The effective interactions between the quasiparticles are expressed in the form of series over multipoles and spin multipoles. It is important that the interaction between the quasiparticles is represented in a separable form, which was first introduced by Yamaguchi.[6] A separable interaction of finite rank $n_{\max}>1$ is used in the cases in which the results of the calculations are more sensitive to the radial dependence of the forces as compared with the calculation of the structure of complex nuclei in the QPNM. It can be assumed that separable interactions of finite rank between the quasiparticles do not restrict the accuracy of the calculations.

We introduce a separable interaction of finite rank for deformed nuclei. For example we consider a central spin-independent interaction $V(|\mathbf{r}_1-\mathbf{r}_2|)$ and expand it with respect to multipoles:

$$V(|\mathbf{r}_1-\mathbf{r}_2|)=\sum_\lambda R^\lambda(r_1,r_2)\frac{4\pi}{2\lambda+1}\sum_{\mu=-\lambda}^{\lambda}\times(-1)^\mu Y_{\lambda\mu}(\theta_1\psi_1)Y_{\lambda-\mu}(\theta_2\psi_2).$$

We represent the radial part $R^\lambda(r_1,r_2)$ in the form

$$R^\lambda(r_1,r_2)=\sum_{n=1}^{n_{\max}}\zeta_n R_n^\lambda(r_1)R_n^\lambda(r_2). \tag{2.2}$$

Most calculations in the QPNM are made with a simple separable interaction

$$R^\lambda(r_1,r_2)=\kappa^\lambda R^\lambda(r_1)R^\lambda(r_2). \tag{2.2'}$$

We transform the initial Hamiltonian of the QPNM. To this end we make a Bogolyubov canonical transformation

$$a_{q\sigma}=u_q\alpha_{q\sigma}+\sigma v_q\alpha^+_{q-\sigma}, \tag{2.3}$$

in order to go over from the particle operators $a_{q\sigma}$ and $a^+_{q\sigma}$ to the quasiparticle operators $\alpha_{q\sigma}$ and $\alpha^+_{q\sigma}$. We introduce phonon operators of two types. If we take into account only interactions of electric type, then the phonon creation operator has the following standard form:

$$Q^+_{Ki_1\sigma}=\tfrac{1}{2}\sum_{q_1q_2}\{\psi^{Ki_1}_{q_1q_2}A^+(q_1q_2;K\sigma)-\phi^{Ki_1}_{q_1q_2}A(q_1q_2;K-\sigma)\}. \tag{2.4}$$

If we take into account electric and magnetic interactions, then we write the phonon operator[5,7] in the form

$$Q^+_{Ki_1\sigma}=\frac{1}{2\sqrt{2}}\sum_{q_1q_2}\{\psi^{Ki_1}_{q_1q_2}(1+i\sigma)[\widetilde{A}^+(q_1q_2;K\sigma)-\chi(q_1q_2)\bar{A}^+(q_1q_2;K\sigma)]-\phi^{Ki_1}_{q_1q_2}(1-i\sigma)[\widetilde{A}(q_1q_2;K-\sigma)+\chi(q_1q_2)\bar{A}(q_1q_2;K-\sigma)]\}. \tag{2.5}$$

The operator (2.5) combines electric and magnetic parts; the coefficients of the electric part are real and of the magnetic part imaginary. This form of the operator is more convenient than the one given earlier in Ref. 4. Here $i=1$, 2, 3,... is the number of the solution of the secular RPA equation and $\psi^{Ki_1}_{q_1q_2}=\psi^{Ki_1}_{q_2q_1}$, $\phi^{Ki_1}_{q_1q_2}=\phi^{Ki_1}_{q_2q_1}$. The quantum numbers of the single-particle states are denoted by $q\sigma$, where $\sigma=\pm1$, $q=K^\pi$ and by the asymptotic quantum number $Nn_z\Lambda\uparrow$ for $K=\Lambda+1/2$ and $Nn_z\Lambda\downarrow$ for $K=\Lambda-1/2$. States that differ in the signs of σ are conjugate with respect to the time reversal operation. The operators $\widetilde{A}(q_1q_2;K\sigma)$ and $\bar{A}(q_1q_2;K\sigma)$ are given in Appendix 1, and

$$\chi(q_1q_2)\bar{A}(q_1q_2;K\sigma)=-\chi(q_2q_1)\bar{A}(q_1q_2;K\sigma)=\chi(q_2q_1)\bar{A}(q_2q_1;K\sigma).$$

The one-phonon state in the RPA is described by the wave function

$$Q^+_{Ki_1\sigma}\Psi_0, \tag{2.6}$$

where Ψ_0 is the wave function of the ground state of the even–even nucleus, defined as the phonon vacuum. The normalization condition of the wave function (2.6) is

$$\frac{1+\delta_{K0}}{2}\sum_{q_1q_2}[(\psi^{Ki_1}_{q_1q_2})^2-(\phi^{Ki_1}_{q_1q_2})^2]=1. \tag{2.7}$$

It is easy to show that the phonon operators $Q^+_{Ki_1\sigma}$ and $Q_{Ki_1\sigma}$ in the form (2.5) satisfy the conditions that are usually imposed on RPA phonons.

By means of the expressions (2.4), (2.5), and others, and making various manipulations, we transform the QPNM Hamiltonian to

$$H_{\rm QPNM}=\sum_{q\sigma}\varepsilon_q\alpha^+_{q\sigma}\alpha_{q\sigma}+H_v+H_{vq}, \tag{2.8}$$

where the first two terms describe the quasiparticles and phonons, and H_{vq} describes the interaction of the quasiparticles with the phonons. They have the form

$$H_v=H_v^{00}+\sum_\lambda H_v^{\lambda0}+\sum_K H_v^K, \tag{2.9}$$

$$H_v^{00}=-\frac{1}{2}\sum_{\tau i_1i_2}\sum_{q_1q_2}{}^\tau G_\tau[(u^2_{q_1}-v^2_{q_1})(u^2_{q_2}-v^2_{q_2})g^{20i_1}_{q_1q_1}g^{20i_2}_{q_2q_2}+w^{20i_1}_{q_1q_1}w^{20i_2}_{q_2q_2}]Q^+_{20i_1}Q_{20i_2}, \tag{2.10}$$

$$H_v^{\lambda0}=\sum_{n=1}^{n_{\max}}\sum_{\tau i_1i_2}\Bigg\{\sum_{\rho=\pm1}[(\kappa_0^{\lambda0}+\rho\kappa_1^{\lambda0})D^{\lambda0i_1}_{n\tau}D^{\lambda0i_2}_{n\rho\tau}-(\kappa_0^{\lambda\lambda0}+\rho\kappa_1^{\lambda\lambda0})D^{\lambda\lambda0i_1}_{n\tau}D^{\lambda\lambda0i_2}_{n\rho\tau}]+G^{\lambda0}[D^{\lambda0i_1}_{gn\tau}D^{\lambda0i_2}_{gn\tau}+D^{\lambda0i_1}_{wn\tau}D^{\lambda0i_2}_{wn\tau}]+G^{\lambda\lambda0}[D^{\lambda\lambda0i_1}_{gn\tau}D^{\lambda\lambda0i_2}_{gn\tau}$$

$$+D_{wn\tau}^{\lambda\lambda 0i_1}D_{wn\tau}^{\lambda\lambda 0i_2}]\Big\}Q_{\lambda 0i_1}^{+}Q_{\lambda 0i_2}, \quad (2.11)$$

$$H_v^K=-\sum_{i_1i_2}W_{i_1i_2}^K Q_{Ki_1\sigma}^{+}Q_{Ki_2\sigma}, \quad (2.12)$$

$$W_{i_1i_2}^K=W_{i_1i_2}^{KE}+W_{i_1i_2}^{KM}+W_{i_1i_2}^{KT}, \quad (2.13)$$

$$W_{i_1i_2}^{KE}=\sum_{\lambda}W_{i_1i_2}^{\lambda K}$$

$$=\frac{1}{4}\sum_{\lambda\tau}\sum_{n=1}^{n_{\max}}\Big\{\sum_{\rho=\pm1}\Big[(\kappa_0^{\lambda K}+\rho\kappa_1^{\lambda K})D_{n\tau}^{\lambda Ki_1}D_{n\rho\tau}^{\lambda Ki_2}$$
$$-(\kappa_0^{\lambda\lambda K}+\rho\kappa_1^{\lambda\lambda K})D_{n\tau}^{\lambda\lambda Ki_1}D_{n\rho\tau}^{\lambda\lambda Ki_2}\Big]$$
$$+G^{\lambda K}\Big(D_{ng\tau}^{\lambda Ki_1}D_{ng\tau}^{\lambda Ki_2}+D_{nw\tau}^{\lambda Ki_1}D_{nw\tau}^{\lambda Ki_2}\Big)$$
$$+G^{\lambda\lambda K}\Big(D_{ng\tau}^{\lambda\lambda Ki_1}D_{ng\tau}^{\lambda\lambda Ki_2}+D_{nw\tau}^{\lambda\lambda Ki_1}D_{nw\tau}^{\lambda\lambda Ki_2}\Big)\Big\}, \quad (2.14)$$

$$W_{i_1i_2}^{KM}=\tfrac{1}{4}\sum_{L\tau}\sum_{\lambda'=L\pm1}\sum_{n=1}^{n_{\max}}\sum_{\rho=\pm1}\Big[(\kappa_0^{\lambda' LK}$$
$$+\rho\kappa_1^{\lambda' LK})D_{n\tau}^{\lambda' LKi_1}D_{n\rho\tau}^{\lambda' LKi_2}$$
$$+G^{\lambda' LK}\Big(D_{ng\tau}^{\lambda' LKi_1}D_{ng\tau}^{\lambda' LKi_2}+D_{nw\tau}^{\lambda' LKi_1}D_{nw\tau}^{\lambda' LKi_2}\Big)\Big], \quad (2.15)$$

$$W_{i_1i_2}^{KT}=\tfrac{1}{2}\sum_{L\tau}\sum_{n=1}^{n_{\max}}\sum_{\rho=\pm1}(\kappa_{T0}^{LK}$$
$$+\rho\kappa_{T1}^{LK})D_{n\tau}^{L-1LKi_1}D_{n\rho\tau}^{L+1LKi_2}, \quad (2.16)$$

$$H_{vq}=H_{vq}^{00}+\sum_{\lambda}H_{vq}^{\lambda 0}+\sum_{K}H_{vq}^{K}, \quad (2.17)$$

$$H_{vq}^{00}=-\sum_{\tau i_1}G_\tau\sum_{q_1q_2}{}^{\tau}(u_{q_1}^2-v_{q_1}^2)u_{q_2}v_{q_2}\Big\{\Big(\psi_{q_1q_1}^{20i_1}Q_{20i_1}^{+}$$
$$+\phi_{q_1q_1}^{20i_1}Q_{20i_1}\Big)\sum_{\sigma}\alpha_{q\sigma}^{+}\alpha_{q\sigma}+\text{h.c.}\Big\}, \quad (2.18)$$

$$H_{vq}^{\lambda 0}=-\tfrac{1}{2}\sum_{\tau i_1}\sum_{n=1}^{n_{\max}}\sum_{q_1q_2}{}^{\tau}V_{n\tau}^{\lambda 0i_1}(q_1q_2)\{(Q_{\lambda 0i_1}^{+}$$
$$+Q_{\lambda 0i_1})B(q_1q_2;$$
$$K=0)+\text{h.c.}\}, \quad (2.19)$$

$$H_{vq}^{K}=-\frac{1}{4}\sum_{n=1}^{n_{\max}}\Big(\sum_{\lambda}H_{vq}^{\lambda K}+\sum_{L,\lambda'=L\pm1}H_{vq}^{\lambda' LK}$$
$$+\sum_{L}H_{vq}^{LKT}\Big), \quad (2.20)$$

$$H_{vq}^{\lambda K}=\sum_{i_1\tau\sigma}\sum_{q_1q_2}{}^{\tau}V_{n\tau}^{\lambda Ki_1}(q_1q_2)\Big[\frac{(1-i\sigma)}{\sqrt{2}}(Q_{Ki_1\sigma}^{+}$$
$$+Q_{Ki_1-\sigma})\mathfrak{B}(q_1q_2;K-\sigma)+\text{h.c.}\Big], \quad (2.21)$$

$$V_{n\tau}^{\lambda Ki_1}(q_1q_2)=f_n^{\lambda K}(q_1q_2)\Big[\sum_{\rho=\pm1}(\kappa_0^{\lambda K}+\rho\kappa_1^{\lambda K})v_{q_1q_2}^{(-)}D_{n\rho\tau}^{\lambda Ki_1}$$
$$-G^{\lambda K}u_{q_1q_2}^{(+)}D_{ng\tau}^{\lambda Ki_1}\Big]$$
$$+f_n^{\lambda\lambda K}(q_1q_2)\sum_{\rho=\pm1}\Big(\kappa_0^{\lambda\lambda K}$$
$$+\rho\kappa_1^{\lambda\lambda K}\Big)v_{q_1q_2}^{(+)}D_{n\rho\tau}^{\lambda\lambda Ki_1}, \quad (2.22)$$

$$H_{vq}^{\lambda' LK}=\sum_{i_1\tau\sigma}\sum_{q_1q_2}{}^{\tau}f_n^{\lambda' LK}(q_1q_2)v_{q_1q_2}^{(+)}\Big(\kappa_0^{\lambda' LK}$$
$$+\rho\kappa_1^{\lambda' LK}\Big)D_{n\rho\tau}^{\lambda' LKi_1}\Big[\frac{(\sigma-i)}{\sqrt{2}}(Q_{Ki_1\sigma}^{(+)}$$
$$-Q_{Ki_1-\sigma})\mathfrak{B}(q_1q_2;K-\sigma)+\text{h.c.}\Big], \quad (2.23)$$

$$H_{vq}^{LKT}=\sum_{i_1\tau\sigma}\mathrm{i}\sum_{q_1q_2}{}^{\tau}(\kappa_{T0}^{LK}+\rho\kappa_{T1}^{LK})$$
$$\times\Big[D_{n\rho\tau}^{L-1LKi_1}f_n^{L+1LK}(q_1q_2)$$
$$+D_{n\rho\tau}^{L+1LKi_2}f_n^{L-1LK}(q_1q_2)\Big]v_{q_1q_2}^{(+)}\Big[(Q_{Ki_1\sigma}^{+}$$
$$-Q_{Ki_1-\sigma})B(q_1q_2;K-\sigma)+\text{h.c.}\Big], \quad (2.24)$$

where

$$D_{n\tau}^{\lambda Ki_1}=\sum_{q_1q_2}{}^{\tau}f_n^{\lambda K}(q_1q_2)u_{q_1q_2}^{(+)}g_{q_1q_2}^{Ki_1}, \quad (2.25)$$

$$D_{ng\tau}^{\lambda Ki_1}=\sum_{q_1q_2}{}^{\tau}f_n^{\lambda K}(q_1q_2)v_{q_1q_2}^{(-)}g_{q_1q_2}^{Ki_1}, \quad (2.26)$$

$$D_{nw\tau}^{\lambda Ki_1}=\sum_{q_1q_2}{}^{\tau}f_n^{\lambda K}(q_1q_2)v_{q_1q_2}^{(+)}w_{q_1q_2}^{Ki_1}, \quad (2.27)$$

$$D_{n\tau}^{\lambda' LKi_1}=\sum_{q_1q_2}{}^{\tau}f_n^{\lambda' LK}(q_1q_2)u_{q_1q_2}^{(-)}w_{q_1q_2}^{Ki_1}\chi(q_1q_2) \quad (2.28)$$

for $\lambda'=L$, $L\pm1$. In the case of simple separable interactions $n_{\max}=1$, the index n in the functions (2.25)–(2.28) is absent and the matrix elements have the form (A9). Here ε_q is the energy of a quasiparticle with monopole pairing; $g_{q_1q_2}^{Ki_1}=\psi_{q_1q_2}^{Ki_1}+\phi_{q_1q_2}^{Ki_1}$, $w_{q_1q_2}^{Ki_1}=\psi_{q_1q_2}^{Ki_1}-\phi_{q_1q_2}^{Ki_1}$. The operators $B(q_1q_2;K\sigma)$ and $\mathfrak{B}(q_1q_2;K\sigma)$, and also the matrix elements of the multipole and spin–multipole operators $f_n^{\lambda K}(q_1q_2)$ and $f_n^{L\pm1KL}(q_1q_2)$ are given in Appendix 1. Further $u_{q_1q_2}^{(\pm)}=u_{q_1}v_{q_2}\pm u_{q_2}v_{q_1}$, $v_{q_1q_2}^{(\pm)}=u_{q_1}u_{q_2}\pm v_{q_1}v_{q_2}$. Summation over the single-particle states of the neutron and proton systems are denoted as $\Sigma_{q_1q_2}^{\tau}$ for $\tau=n$ or $\tau=p$, respectively. Replacement of τ by $-\tau$ means replacement of n by p:

$$\sum_{\tau}A(\tau)B(-\tau)=A(p)B(n)+A(n)B(p),$$

$$\sum_{\rho=\pm 1} A(\tau)B(\rho\tau)=A(\tau)B(\tau)+A(\tau)B(-\tau).$$

The constants of the monopole neutron, $\tau=n$, and proton, $\tau=p$, pairing are denoted by G_τ. The isoscalar and isovector constants of the *ph* multipole interaction are denoted by $\kappa_0^{\lambda K}$ and $\kappa_1^{\lambda K}$, and $G^{\lambda K}=G_0^{\lambda K}+G_1^{\lambda K}$ is the constant of the *pp* multipole interaction; $\kappa_0^{\lambda' LK}$, $\kappa_1^{\lambda' LK}$ and κ_{T0}^{LK}, κ_{T1}^{LK} are the constants of the isoscalar and isovector *ph* spin-multipole and tensor interactions, and $G^{\lambda'}LK$ is the constant of the *pp* spin-multipole interaction.

Most calculations of the structure of excited states and the $B(E\lambda)$ values have been made with the phonon operator (2.4) and simple ($n_{\max}=1$) multipole interactions with H_{vq}^K in the form

$$H_{vq}^K=\sum_\lambda H_{vq}^{\lambda K}$$
$$=-\frac{1}{4}\sum_{i_1\tau\sigma}\sum_\lambda\sum_{\rho=\pm 1}\sum_{q_1q_2}{}^{\tau} f^{\lambda K}(q_1q_2)v_{q_1q_2}^{(-)}(\kappa_0^{\lambda K}$$
$$+\rho\kappa_1^{\lambda K})D_{\rho\tau}^{\lambda K i_1}[(Q_{Ki_1\sigma}^{+}+Q_{Ki_1-\sigma})B(q_1q_2;$$
$$K-\sigma)+\text{h.c.}], \tag{2.29}$$

and the probabilities of $M\lambda$ transitions are calculated in Ref. 7 with the phonon operator (2.5) but without the *pp* spin-multipole interaction.

Calculations in the QPNM are made in four stages. The first stage consists of the calculation of the single-particle energies and wave functions of the Woods–Saxon potential. The parameters of this potential are fixed in four steps: 1) the single-particle energies and wave functions are calculated with certain parameters of the potential; 2) the equilibrium shape of the nucleus is calculated by Strutinsky's shell-correction method, and in this way the parameters of the quadrupole, β_2, and hexadecupole, β_4, deformations are also fixed; 3) the phonons are calculated in the RPA; 4) the wave function of an odd nucleus is taken in the form of a sum of a one-quasiparticle component and a quasiparticle–phonon component; the quasiparticle–phonon interactions are taken into account and the calculated energies and wave functions of the nonrotational states of odd nuclei are compared with experimental data. If sufficiently good agreement with the experimental data is not obtained, then the parameters of the Woods–Saxon potential are changed, and new calculations of all four steps are made. This procedure is repeated until a sufficiently good description of the experimental data on the low-lying nonrotational states in the odd nuclei has been obtained. One could no doubt use a different form of the potential of the average field or calculate the energies and wave functions of the single-particle states by the Hartree–Fock method in order to use them in calculations in the QPNM.

In the second stage of Bogolyubov canonical transformation is used to go over from the particle operators $a_{q\sigma}$ and $a_{q\sigma}^{+}$ to the quasiparticle operators $\alpha_{q\sigma}$ and $\alpha_{q\sigma}^{+}$, and calculations are then made in the framework of the model of independent quasiparticles. Most calculations are made with monopole pairing. If a monopole pairing with constant G_τ and a quadrupole pairing with constant G^{20} are present simultaneously, then the condition of elimination of 0^+ ghost states yields the equations[8]

$$1=\frac{G_\tau}{2}\sum_q{}^{\tau}\frac{C_\tau+f^{20}(qq)C_{2\tau}}{C_\tau\varepsilon_q}, \tag{2.30}$$

$$1=G^{20}\left\{\sum_q{}^{\tau}\frac{f^{20}(qq)C_\tau}{2C_{2\tau}\varepsilon_q}+\sum_{qq'}{}^{\tau}\frac{(f^{20}(qq')v_{qq'}^{(+)})^2}{\varepsilon_q+\varepsilon_{q'}}\right\}, \tag{2.31}$$

and

$$N_\tau=\sum_q{}^{\tau}\left[1-\frac{\xi(q)}{\varepsilon_q}\right]. \tag{2.32}$$

Ignoring the nondiagonal matrix elements $f^{20}(qq')$ in Eq. (2.31), we arrive at the equations obtained earlier in Refs. 9 and 10. Here

$$\varepsilon_q=[\Delta_q^2+\xi^2(q)]^{1/2},\quad \xi(q)=E(q)-\lambda_\tau,$$
$$\Delta_q=C_\tau+f^{20}(qq)C_{2\tau},\quad C_\tau=G_\tau\sum_q{}^{\tau}u_qv_q, \tag{2.33}$$
$$C_{2\tau}=G^{20}\sum_q{}^{\tau} f^{20}(qq)u_qv_q,$$

where $E(q)$ is the single-particle energy, and λ_τ is the chemical potential. The energies of the two-quasiparticle states are calculated with allowance for the blocking effect (see Ref. 1).

After this the RPA phonons (2.4) or (2.5) are introduced and the RPA secular equation is solved. In the QPNM the one-phonon states (2.6) with the operator (2.4) are used as basis. Thus the third stage consists of calculating a one-phonon basis. In actual calculations of low-lying states the one-phonon basis usually consists of ten ($i_1=1, 2,...,10$) phonons of each multipolarity: quadrupole ($\lambda_\mu=20$, 21, 22), octupole ($\lambda_\mu=30$, 31, 32, 33), and hexadecapole ($\lambda_\mu=43$, 44). The states with energy above 3 MeV are calculated with a large phonon basis with $\lambda>4$ and 20 phonons of each multipolarity. The phonon space corresponds to the complete space of two-quasiparticle states in even–even deformed nuclei.

As a result of the transformations, the QPNM Hamiltonian is reduced to the form (2.8). In the fourth stage the interaction of the quasiparticles with the phonons is taken into account. The wave function of the excited states is represented as a series with respect to the number of phonon operators; in odd nuclei the wave function consists of sums of single-quasiparticle, quasiparticle–phonon, etc., terms. The approximation consists of the truncation of this series. In the calculations the Pauli principle is taken into account by means of exact commutation relations between the operators of the phonons and quasiparticles with the phonons. The strength-function method is used to calculate the characteristics of the highly excited states. Using one of the forms of the strength-function method, one can directly calculate the reduced transition probabilities, spectroscopic factors, transition densities, cross sections, and other characteristics without solving the corresponding secular equations.

The interaction of the quasiparticles with the phonons leads to fragmentation of the one-quasiparticle, one-phonon, quasiparticle–phonon, two-phonon, and other states. The quasiparticle–phonon interaction is responsible for the increasing complexity of the structure of the nuclear states with increasing excitation energy.

3. RPA EQUATIONS

3.1. RPA equations for 0^+ states

We shall give the RPA equations for different cases. The one-phonon states form the basis of the QPNM. We therefore devote great attention to the solution of the RPA equations. In the RPA the interactions between the quasiparticles in the ground and excited states are taken into account. The ground-state wave function Ψ_0 of an even–even nucleus is defined as the vacuum relative to the various phonons. Besides quasiparticleless terms, it contains four-, eight-, and more-quasiparticle terms (see Ref. 1). The RPA is valid when the mean number $\langle\alpha^+_{q\sigma}\alpha_{q\sigma}\rangle$ of the quasiparticles in the ground state is small.

Excited $K^\pi=0^+$ states occupy a special place in nuclear theory. The impression is created that many difficulties of the theory are concentrated on them. The wave functions of the 0^+ states are determined by the pairing and quadrupole and possibly hexadecapole interactions. Besides one-phonon terms, they contain two-phonon terms and, at higher excitation energies, many-phonon terms. In the RPA description of the 0^+ states, the ghost states that arise from the conservation of the number of neutrons and protons on the average must be eliminated.

The $K=0$ states are described by means of σ-independent phonon operators in the form

$$Q^+_{\lambda 0 i_1}=\frac{1}{2}\sum_{q_1q_2}\left[\psi^{\lambda 0 i_1}_{q_1q_2}A^+(q_1q_2;K=0)-\phi^{\lambda 0 i_1}_{q_1q_2}A(q_1q_2;K=0)\right], \tag{3.1}$$

where

$$A^+(q_1q_2;K=0)=\sum_\sigma \sigma\alpha^+_{q_1\sigma}\alpha^+_{q_2-\sigma}.$$

The normalization condition for the wave function

$$Q^+_{\lambda 0 i_1}\Psi_0 \tag{3.2}$$

has the form

$$\sum_{q_1q_2}\left[\psi^{\lambda 0 i_1}_{q_1q_2}\psi^{\lambda 0 i_2}_{q_1q_2}-\phi^{\lambda 0 i_1}_{q_1q_2}\phi^{\lambda 0 i_2}_{q_1q_2}\right]=\delta_{i_1i_2}, \tag{3.3}$$

and

$$A(q_1q_2;K=0)=2\sum_{i_1}\left[\psi^{\lambda 0 i_1}_{q_1q_2}Q_{\lambda 0 i_1}+\phi^{\lambda 0 i_1}_{q_1q_2}Q^+_{\lambda 0 i_1}\right].$$

For the description of the 0^+ states, we use simple separable forces, $n_{\max}=1$, and ignore the spin-multipole and hexadecapole forces. We use the Hamiltonian

$$H_{\rm RPA}(0^+)=\sum_{q\sigma}\varepsilon_q\alpha^+_{q\sigma}\alpha_{q\sigma}-\sum_{\tau i_1 i_2}\Bigg\{\frac{1}{2}\sum_{q_1q_2}{}^{\tau}\, G_\tau[(u^2_{q_1}-v^2_{q_1})(u^2_{q_2}-v^2_{q_2})g^{20i_1}_{q_1q_1}g^{20i_2}_{q_2q_2}+w^{20i_1}_{q_1q_1}w^{20i_2}_{q_2q_2}]+\sum_{\rho=\pm1}(\kappa_0^{20}+\rho\kappa_1^{20})D^{20i_1}_{\tau}D^{20i_2}_{\rho\tau}+G^{20}\Big[D^{20i_1}_{g\tau}D^{20i_2}_{g\tau}+D^{20i_1}_{w\tau}D^{20i_2}_{w\tau}\Big]\Bigg\}. \tag{3.4}$$

We calculate the mean value of $H_{\rm RPA}(0^+)$ with respect to the state (3.2), and to find the energies ω_{20i_1} of the one-phonon states we use a variational principle in the form

$$\delta\left\{\langle Q_{20i_1}H_{\rm RPA}(0^+)Q^+_{20i_1}\rangle-\frac{\omega_{20i_1}}{2}\left[\sum_{q_1q_2}g^{20i_1}_{q_1q_2}w^{20i_1}_{q_1q_2}-2\right]\right\}=0, \tag{3.5}$$

where ω_{20i_1} plays the role of a Lagrangian multiplier, and the variations $\delta g^{20i_1}_{q_1q_2}$ and $\delta w^{20i_1}_{q_1q_2}$ are treated as independent. We obtain the equations

$$\varepsilon_{q_1q_2}g^{20i_1}_{q_1q_2}-\omega_{20i_1}w^{20i_1}_{q_1q_2}-G_\tau\delta_{q_1q_2}(u^2_{q_1}-v^2_{q_1})d^{i_1}_{g\tau}-2G^{20}f^{20}(q_1q_2)v^{(-)}_{q_1q_2}D^{20i_1}_{\tau}-2\sum_{\rho=\pm1}(\kappa_0^{20}+\rho\kappa_1^{20})f^{20}(q_1q_2)u^{(+)}_{q_1q_2}D^{20i_1}_{\rho\tau}=0, \tag{3.6}$$

$$\varepsilon_{q_1q_2}w^{20i_1}_{q_1q_2}-\omega_{20i_1}g^{20i_1}_{q_1q_2}-G_\tau\delta_{q_1q_2}d^{i_1}_{w\tau}-2G^{20}f^{20}(q_1q_2)v^{(+)}_{q_1q_2}D^{20i_1}_{\omega\tau}=0, \tag{3.7}$$

where

$$\varepsilon_{q_1q_2}=\varepsilon_{q_1}+\varepsilon_{q_2},\quad d^{i_1}_{g\tau}=\sum_q{}^{\tau}\frac{\xi(q)}{\varepsilon_q}g^{20i_1}_{qq},\quad d^{i_1}_{w\tau}=\sum_q{}^{\tau}w^{20i_1}_{qq}. \tag{3.8}$$

From Eqs. (3.6) and (3.7) we find $g^{20i_1}_{q_1q_2}$ and $w^{20i_1}_{q_1q_2}$, which are substituted in the functions $D^{20i_1}_{\tau}$, $D^{20i_1}_{g\tau}$, $d^{i_1}_{g\tau}$ and $d^{i_1}_{w\tau}$. The corresponding system of equations is given in Ref. 8. The ghost states are then eliminated. From the condition of elimination of the ghost states with $\omega_{200}=0$ we obtain Eqs. (2.30)–(2.32), which describe monopole and quadrupole pairings. In this case ε_q is given by Eq. (2.33). As a result we obtain the system of equations

$$\sum_{\rho=\pm1}(\kappa_0^{20}+\rho\kappa_1^{20})X^{20i_1}_{\rho\tau}D^{20i_1}_{\rho\tau}-D^{20i_1}_{\tau}+G^{20}X^{20i_1}_{\varepsilon\tau}D^{20i_1}_{g\tau}+G^{20}_{\tau}X^{20i_1}_{\omega\tau}D^{20i_1}_{w\tau}+G_\tau V^{i_1}_{\tau}d^{i_1}_{g\tau}+G_\tau W^{20i_1}_{\tau}d^{i_1}_{w\tau}=0, \tag{3.9}$$

$$\sum_{\rho=\pm1}(\kappa_0^{20}+\rho\kappa_1^{20})X^{20i_1}_{\varepsilon\tau}D^{20i_1}_{\rho\tau}+[G^{20}X^{20i_1-}_{\tau}-1]D^{20i_1}_{g\tau}+G^{20}X^{20i_1}_{2\tau}D^{20i_1}_{w\tau}+G_\tau V^{i_1}_{\xi\tau}d^{i_1}_{g\tau}+G_\tau W^{20i_1}_{3\tau}d^{i_1}_{w\tau}=0, \tag{3.10}$$

$$\sum_{\rho=\pm1}(\kappa_0^{20}+\rho\kappa_1^{20})X^{20i_1}_{\omega\tau}D^{20i_1}_{\rho\tau}+G^{20}X^{20i_1}_{2\tau}D^{20i_1}_{g\tau}$$

$$+[G^{20}X_{\tau}^{20i_1+}-1]D_{\omega\tau}^{20i_1}+G_\tau V_{\omega\tau}^{i_1}d_{g\tau}^{i_1}+G_\tau W_{2\tau}^{20i_1}d_{\omega\tau}^{i_1}=0, \tag{3.11}$$

$$\sum_{\rho=\pm 1}(\kappa_0^{20}+\rho\kappa_1^{20})V_{\tau}^{i_1}D_{\rho\tau}^{20i_1}+G^{20}V_{\xi\tau}^{i_1}D_{g\tau}^{20i_1}+G^{20}V_{\omega\tau}^{i_1}D_{\omega\tau}^{20i_1}$$
$$+[G_\tau\mathfrak{L}_{\tau}^{i_1}-1]d_{g\tau}^{i_1}+G_\tau\mathfrak{L}_{2\tau}^{i_1}d_{\omega\tau}^{i_1}=0, \tag{3.12}$$

$$\sum_{\rho=\pm 1}(\kappa_0^{20}+\rho\kappa_1^{20})W_{1\tau}^{20i_1}D_{\rho\tau}^{20i_1}+G^{20}W_{3\tau}^{20i_1}D_{g\tau}^{20i_1}$$
$$+G^{20}W_{2\tau}^{20i_1}D_{\omega\tau}^{20i_1}+G_\tau\mathfrak{L}_{2\tau}^{i_1}d_{g\tau}^{i_1}+G_\tau W_{\tau}^{20i_1}d_{\omega\tau}^{i_1}=0. \tag{3.13}$$

To find the energies ω_{20i} and wave functions of the 0^+ states, it is necessary to solve the system of 10 equations (3.9)–(3.13) for $\tau=p$ and $\tau=n$ with allowance for the normalization condition (3.3). Here

$$X_{\tau}^{20i_1}=(1+\delta_{K0})\sum_{q_1q_2}^{\tau}\frac{[f^{20}(q_1q_2)u_{q_1q_2}^{(+)}]^2\varepsilon_{q_1q_2}}{\varepsilon_{q_1q_2}^2-\omega_{20i_1}^2},$$

$$X_{\tau}^{20i_1\pm}=(1+\delta_{K0})\sum_{q_1q_2}^{\tau}\frac{[f^{20}(q_1q_2)v_{q_1q_2}^{(\pm)}]^2\varepsilon_{q_1q_2}}{\varepsilon_{q_1q_2}^2-\omega_{20i_1}^2}, \tag{3.14}$$

$$X_{2\tau}^{20i_1}=(1+\delta_{K0})\sum_{q_1q_2}^{\tau}\frac{[f^{20}(q_1q_2)]^2v_{q_1q_2}^{(+)}v_{q_1q_2}^{(-)}\omega_{20i_1}}{\varepsilon_{q_1q_2}^2-\omega_{20i_1}^2},$$

$$X_{\omega\tau}^{20i_1}=\sum_{q_1q_2}^{\tau}\frac{[f^{20}(q_1q_2)]^2u_{q_1q_2}^{(+)}v_{q_1q_2}^{(+)}\omega_{20i_1}}{\varepsilon_{q_1q_2}^2-\omega_{20i_1}^2}, \tag{3.15}$$

$$X_{\varepsilon\tau}^{20i_1}=\sum_{q_1q_2}^{\tau}\frac{[f^{20}(q_1q_2)]^2u_{q_1q_2}^{(+)}v_{q_1q_2}^{(-)}\varepsilon_{q_1q_2}}{\varepsilon_{q_1q_2}^2-\omega_{20i_1}^2},$$

$$W_{\tau}^{20i_1}=\sum_{q_1q_2}^{\tau}\frac{[f^{20}(q_1q_2)v_{q_1q_2}^{(+)}]^2C_{2\tau}^2}{\varepsilon_{q_1q_2}(\varepsilon_{q_1q_2}^2-\omega_{20i_1}^2)}+\sum_{q}^{\tau}\frac{C_\tau^2+2f^{20}(qq)C_\tau C_{2\tau}}{2\varepsilon_q(4\varepsilon_q^2-\omega_{20i_1}^2)},$$

$$W_{1\tau}^{20i_1}=\sum_{q_1q_2}^{\tau}\frac{[f^{20}(q_1q_2)]^2u_{q_1q_2}^{(+)}v_{q_1q_2}^{(-)}C_{2\tau}}{\varepsilon_{q_1q_2}^2-\omega_{20i_1}^2}+\sum_{q}^{\tau}\frac{f^{20}(qq)C_\tau^2}{\varepsilon_q(4\varepsilon_q^2-\omega_{20i_1}^2)},$$

$$W_{2\tau}^{20i_1}=\sum_{q_1q_2}^{\tau}\frac{[f^{20}(q_1q_2)v_{q_1q_2}^{(+)}]^2C_{2\tau}^2\omega_{20i_1}}{\varepsilon_{q_1q_2}(\varepsilon_{q_1q_2}^2-\omega_{20i_1}^2)}+\sum_{q}^{\tau}\frac{f^{20}(qq)C_\tau\omega_{20i_1}}{2\varepsilon_q(4\varepsilon_q^2-\omega_{20i_1}^2)}, \tag{3.16}$$

$$W_{3\tau}^{20i_1}=\sum_{q_1q_2}^{\tau}\frac{[f^{20}(q_1q_2)]^2v_{q_1q_2}^{(-)}v_{q_1q_2}^{(+)}C_{2\tau}}{\varepsilon_{q_1q_2}^2-\omega_{20i_1}^2}+\sum_{q}^{\tau}\frac{f^{20}(qq)\xi(q)C_\tau}{\varepsilon_q(4\varepsilon_q^2-\omega_{20i_1}^2)},$$

$$V_{\xi\tau}^{i_1}=\sum_{q}^{\tau}\frac{f^{20}(qq)2\xi^2(q)}{\varepsilon_q(4\varepsilon_q^2-\omega_{20i_1}^2)},\quad V_{\omega\tau}^{i_1}=\sum_{q}^{\tau}\frac{f^{20}(qq)\xi(q)\omega_{20i_1}}{\varepsilon_q(4\varepsilon_q^2-\omega_{20i_1}^2)},\quad V_{\tau}^{i_1}=\sum_{q}^{\tau}\frac{f^{20}(qq)2\xi(q)C_\tau}{\varepsilon_q(4\varepsilon_q^2-\omega_{20i_1}^2)}, \tag{3.17}$$

$$\mathfrak{L}_{\tau}^{i_1}=\sum_{q}^{\tau}\frac{2\xi^2(q)}{\varepsilon_q(4\varepsilon_q^2-\omega_{20i_1}^2)},\quad \mathfrak{L}_{2\tau}^{i_1}=\sum_{q}^{\tau}\frac{[C_\tau+f^{20}(qq)C_{2\tau}]\xi(q)}{\varepsilon_q(4\varepsilon_q^2-\omega_{20i_1}^2)}. \tag{3.18}$$

The determinant of the system of equations (3.9)–(3.13) has rank 10; it is given in Ref. 8. If separable quadrupole interactions of rank $n_{\max}>1$ are taken into account, then the rank of the determinant for finding the energies ω_{20i_1} of the one-phonon 0^+ states will be $4+6n_{\max}$. Our calculations of 0^+ states in Refs. 11–14 were made with $n_{\max}=1$ and radial dependence of the quadrupole forces in the form $R^{\lambda}(r)=\partial V(r)/\partial r$, where $V(r)$ is the central part of the Woods–Saxon potential. Among the solutions of these equations there are no excess ghost solutions. The ghost solutions must be eliminated. If they are not eliminated, then two ghost states are distributed among the lowest 0^+ states.

Investigations[11,12] of the 0^+ states showed that the role of the *pp* interactions is important. With increasing G^{20} the low-lying poles of the RPA secular equation change. As a result the value of $B(E2)$ for excitation of the first $I^{\pi}K_{i_1} = 2^+0_1$ state and the energies of the 0_2^+, 0_3^+, 0_4^+, etc., states are reduced for $G^{20}=\kappa_0^{20}$ compared with $G^{20}=0$. Moreover the wave functions of the 0^+ states are changed. The inclusion of *pp* interactions basically improves the description of the 0^+ states.

Calculations of superdeformed 0^+ states in ^{238}U and ^{240}Pu were made in Ref. 15. The calculated energies of the first excited 0^+ states are close to the experimental data. The probabilities of $E0$ transitions in the superdeformed states are in agreement with the experimental data. The second and some other 0^+ states in ^{238}U and ^{240}Pu belong to the isovector type. Quadrupole pairing plays an important role in the description of the superdeformed states.

3.2. RPA equations for $K^{\pi}\neq 0^+$ states

We give the RPA equations for the energies ω_{Ki_1} and wave functions (2.6) of the one-phonon states with $K^{\pi}\neq 0^+$ in both the general case and for different special cases.

We first consider a fairly general case. We take the phonon operator in the form (2.5) and use the following part of the Hamiltonian (2.8)–(2.10):

$$H_{\rm RPA}(K^\pi\neq 0^+)=\sum_{q\sigma}\varepsilon_q\alpha^+_{q\sigma}\alpha_{q\sigma}+H^K_v, \tag{3.19}$$

ignoring the spin–multipole and tensor pp interactions. We determine the expectation value of (3.19) with respect to the states (2.6) and, using the variational principle, obtain the equations

$$\varepsilon_{q_1q_2}g^{Ki_1}_{q_1q_2}-\omega_{Ki_1}w^{Ki_1}_{q_1q_2}-\sum_{n=1}^{n_{\max}}\sum_\lambda f_n^{\lambda K}(q_1q_2)\Big[u^{(+)}_{q_1q_2}\sum_{\rho=\pm1}\big(\kappa_0^{\lambda K}+\rho\kappa_1^{\lambda K}\big)D^{\lambda Ki_1}_{n\rho\tau}+v^{(-)}_{q_1q_2}G^{\lambda K}D^{\lambda Ki_1}_{ng\tau}\Big]=0, \tag{3.20}$$

$$\varepsilon_{q_1q_2}w^{Ki_1}_{q_1q_2}-\omega_{Ki_1}g^{Ki_1}_{q_1q_2}-\sum_{n=1}^{n_{\max}}\Big\{\sum_\lambda f_n^{\lambda K}(q_1q_2)v^{(+)}_{q_1q_2}G^{\lambda K}D^{\lambda Ki_1}_{nw\tau}+\sum_L\sum_{\rho=\pm1}\Big[\sum_{\lambda'}\big(\kappa_0^{\lambda' LK}+\rho\kappa_1^{\lambda' LK}\big)D^{\lambda' LKi_1}_{n\rho\tau}f_n^{\lambda' LK}(q_1q_2)\times u^{(-)}_{q_1q_2}\chi(q_1q_2)+\big(\kappa^{LK}_{T0}+\rho\kappa^{LK}_{T1}\big)u^{(-)}_{q_1q_2}\chi(q_1q_2)\times\big(f_n^{L-1LK}(q_1q_2)D^{L+1LKi_1}_{n\rho\tau}+f_n^{L+1LK}(q_1q_2)D^{L-1LKi_1}_{n\rho\tau}\big)\Big]\Big\}=0. \tag{3.21}$$

Here

$$\varepsilon_{q_1q_2}=\varepsilon_{q_1}+\varepsilon_{q_2},\quad g^{Ki_1}_{q_1q_2}=\psi^{Ki_1}_{q_1q_2}+\phi^{Ki_1}_{q_1q_2},\quad\text{and}\quad w^{Ki_1}_{q_1q_2}=\psi^{Ki_1}_{q_1q_2}-\phi^{Ki_1}_{q_1q_2}.$$

From Eqs. (3.20) and (3.21) we obtain the functions $g^{Ki_1}_{q_1q_2}$ and $w^{Ki_1}_{q_1q_2}$ and we substitute them in the expressions (2.25)–(2.28) for $D^{\lambda Ki_1}_{n\tau}$, $D^{\lambda Ki_1}_{ng\tau}$, $D^{\lambda Ki_1}_{nw\tau}$ and $D^{\lambda' LKi_1}_{n\tau}$. Bearing in mind that $\tau=n$, $\tau=p$ and $\lambda'=L-1$, L, $L+1$, we obtain a secular equation for the energies of the one-phonon states in the form of the vanishing of a determinant of dimension $6(n_\lambda+n_L)$, where n_λ and n_L are the numbers of terms in the sums over λ and L in Eqs. (3.20) and (3.21). The use of the separable interaction of rank $n_{\max}$ increases the dimension of the determinant by $n_{\max}$ times compared with the simple separable interaction ($n_{\max}=1$). If the spin-multipole terms with $\lambda'=L$ are ignored, we obtain a determinant of dimension $(6n_\lambda+4n_L)n_{\max}$.

Most calculations (Refs. 11–14, 16, and 17) of one-phonon states with $K^\pi\neq 0^+$ have been made with ph and pp interactions of one multipolarity with $n_{\max}=1$. One uses a phonon operator in the form (2.4) and the matrix elements A9) with radial dependence $R^\lambda(r)=\partial V(r)/\partial r$, where $V(r)$ is the central part of the Woods–Saxon potential. The RPA equations for the states of multipolarity λ with K^π, where $\pi=(-1)^\lambda$, have the form

$$\varepsilon_{q_1q_2}g^{\lambda Ki_1}_{q_1q_2}-\omega_{\lambda Ki_1}w^{\lambda Ki_1}_{q_1q_2}-\sum_{\rho=\pm1}\big(\kappa_0^{\lambda K}+\rho\kappa_1^{\lambda K}\big)f^{\lambda K}(q_1q_2)u^{(+)}_{q_1q_2}D^{\lambda Ki_1}_{\rho\tau}-G^{\lambda K}f^{\lambda K}(q_1q_2)v^{(-)}_{q_1q_2}D^{\lambda Ki_1}_{g\tau}=0, \tag{3.22}$$

$$\varepsilon_{q_1q_2}w^{\lambda Ki_1}_{q_1q_2}-\omega_{\lambda Ki_1}g^{\lambda Ki_1}_{q_1q_2}-G^{\lambda K}f^{\lambda K}(q_1q_2)v^{(+)}_{q_1q_2}D^{\lambda Ki_1}_{w\tau}=0. \tag{3.23}$$

From these equations we find $g^{\lambda Ki_1}_{q_1q_2}$ and $w^{\lambda Ki_1}_{q_1q_2}$, which we substitute in the expressions (2.25)–(2.27) for $D^{\lambda Ki_1}_{\tau}$, $D^{\lambda Ki_1}_{g\tau}$ and $D^{\lambda Ki_1}_{w\tau}$. As a result we obtain the system of equations

$$\sum_{\rho=\pm1}\big(\kappa_0^{\lambda K}+\rho\kappa_1^{\lambda K}\big)X^{\lambda Ki_1}_{\rho\tau}D^{\lambda Ki_1}_{\rho\tau}-D^{\lambda Ki_1}_{\tau}+G^{\lambda K}\big[X^{\lambda Ki_1}_{\varepsilon\tau}D^{\lambda Ki_1}_{g\tau}+X^{\lambda Ki_1}_{\omega\tau}D^{\lambda Ki_1}_{w\tau}\big]=0, \tag{3.24}$$

$$\sum_{\rho=\pm1}\big(\kappa_0^{\lambda K}+\rho\kappa_1^{\lambda K}\big)X^{\lambda Ki_1}_{\varepsilon\rho\tau}D^{\lambda Ki_1}_{\rho\tau}+\big[G^{\lambda K}X^{\lambda Ki_1-}_{\tau}-1\big]D^{\lambda Ki_1}_{g\tau}+G^{\lambda K}X^{\lambda Ki_1}_{2\tau}D^{\lambda Ki_1}_{w\tau}=0, \tag{3.25}$$

$$\sum_{\rho=\pm1}\big(\kappa_0^{\lambda K}+\rho\kappa_1^{\lambda K}\big)X^{\lambda Ki_1}_{\omega\rho\tau}D^{\lambda Ki_1}_{\rho\tau}+G^{\lambda K}X^{\lambda Ki_1}_{2\tau}D^{\lambda Ki_1}_{g\tau}+\big[G^{\lambda K}X^{\lambda Ki_1+}_{\tau}-1\big]D^{\lambda Ki_1}_{w\tau}=0. \tag{3.26}$$

The functions X are given by the expressions (3.14) and (3.15), in which 20 is replaced by λK. The energies $\omega_{\lambda Ki_1}$ and wave functions are found by solving Eqs. (2.7) and (3.24)–(3.26). The rank of the determinant of the system is 6.

We give the RPA equations for the multipole interaction with the radial function

$$R^\lambda(r_1r_2)=\frac{\partial V(r_1)}{\partial r_1}\frac{\partial V(r_2)}{\partial r_2}+\zeta\frac{1}{r_0^2}V(r_1)V(r_2).$$

This equation contains a surface part and a density-dependent part; ζ is a new free parameter, and $r_0=1.2$ fm. We denote the matrix elements $\langle q_1|1/r_0V(r)Y_{\lambda K}(\theta,\phi)|q_2\rangle$ by $f_2^{\lambda K}(q_1q_2)$. The connection between a separable multipole interaction and a Skyrme interaction was investigated in Ref. 18. A separable interaction containing a density-dependent part was introduced; in the framework of the RPA it was found to be equivalent to a zero-range Skyrme interaction. Equivalent separable forces can be expressed in terms of transition densities for vibrational states.

We represent the function $W^{KE}_{i_1i_2}$ (2.14) in the form

$$W^{KE}_{i_1i_2}=\frac{1}{4}\sum_\tau\Big\{\sum_{\rho=\pm1}\big(\kappa_0^{\lambda K}+\rho\kappa_1^{\lambda K}\big)\big[D^{\lambda Ki_1}_{\tau}D^{\lambda Ki_2}_{\rho\tau}+\zeta D^{\lambda Ki_1}_{2\tau}D^{\lambda Ki_2}_{2\rho\tau}\big]+G^{\lambda K}\big[D^{\lambda Ki_1}_{g\tau}D^{\lambda Ki_2}_{g\tau}+D^{\lambda Ki_1}_{w\tau}D^{\lambda Ki_2}_{w\tau}+\zeta\big(D^{\lambda Ki_1}_{2g\tau}D^{\lambda Ki_2}_{2g\tau}+D^{\lambda Ki_1}_{2w\tau}D^{\lambda Ki_2}_{w\tau}\big)\big]\Big\}, \tag{3.27}$$

where $D^{\lambda Ki_1}_{2\tau}$, $D^{\lambda Ki_1}_{2g\tau}$, and $D^{\lambda Ki_1}_{2w\tau}$ have matrix element $f_2^{\lambda K}(q_1q_2)$ instead of $f^{\lambda K}(q_1q_2)$. In this case we obtain the RPA equations

$$\varepsilon_{q_1q_2}g_{q_1q_2}^{Ki_1}-\omega_{Ki_1}w_{q_1q_2}^{Ki_1}-\sum_{\rho=\pm1}\left(\kappa_0^{\lambda K}+\rho\kappa_1^{\lambda K}\right)u_{q_1q_2}^{(+)}$$
$$\times\left[f^{\lambda K}(q_1q_2)D_{\rho\tau}^{\lambda Ki_1}+\zeta f_2^{\lambda K}(q_1q_2)D_{2\rho\tau}^{\lambda Ki_2}\right]-G^{\lambda K}v_{q_1q_2}^{(-)}$$
$$\times\left[f^{\lambda K}(q_1q_2)D_{g\tau}^{\lambda Ki_1}+\zeta f_2^{\lambda K}(q_1q_2)D_{2g\tau}^{\lambda Ki_1}\right]=0 \tag{3.28}$$

and

$$\varepsilon_{q_1q_2}w_{q_1q_2}^{Ki_1}-\omega_{Ki_1}g_{q_1q_2}^{Ki_1}-G^{\lambda K}v_{q_1q_2}^{(+)}\left[f^{\lambda K}(q_1q_2)D_{w\tau}^{\lambda Ki_1}\right.$$
$$\left.+\zeta f_2^{\lambda K}(q_1q_2)D_{2w\tau}^{\lambda Ki_1}\right]=0. \tag{3.29}$$

The order of the determinant of the secular equation is 12.

Calculations of $K^\pi=2^+$ states in ^{168}Er using Eqs. (3.28) and (3.29) show that the influence of the density-dependent part of the interaction is small for $\zeta=0.1$–0.2. The quadrupole strength is shifted for $\zeta>0.4$ from the first 2_1^+ state to a state with higher energy, in disagreement with experimental data. This indicates that a separable density-dependent interaction is not so important for the description of the vibrational states. The transition densities are more sensitive to the density-dependent interactions.

The most collective low-lying vibrational states are of quadrupole and octupole type. These states should be described by a phonon operator in the form (2.4) with allowance for *ph* and *pp* multipole interactions. The particle–particle *pp* interactions improve the description of the energies, the $B(E\lambda)$ values, and the structure of these states. The $E1$ transitions from the ground state to $K^\pi=0^-$ or 1^- states should be described with allowance for the *ph* and *pp* octupole interactions, and also the *ph* dipole interactions. To calculate $E4$ or $E5$ transitions to $K^\pi=2^+$ or $K^\pi=0^-$, 1^-, 2^-, 3^- states, the wave functions of these states must be described with allowance for the *ph* and *pp* quadrupole and *ph* hexadecapole interactions or the *ph* and *pp* octupole and *ph* $\lambda=5$ interactions. The RPA equations for such cases have the form

$$\varepsilon_{q_1q_2}g_{q_1q_2}^{Ki_1}-\omega_{Ki_1}w_{q_1q_2}^{Ki_1}-f^{\lambda K}(q_1q_2)\left[u_{q_1q_2}^{(+)}\sum_{\rho=\pm1}\left(\kappa_0^{\lambda K}\right.\right.$$
$$\left.\left.+\rho\kappa_1^{\lambda K}\right)D_{\rho\tau}^{\lambda Ki_1}+v_{q_1q_2}^{(-)}G^{\lambda K}D_{g\tau}^{\lambda Ki_1}\right]$$
$$-f^{\lambda\pm2K}(q_1q_2)u_{q_1q_2}^{(+)}\sum_{\rho=\pm1}\left(\kappa_0^{\lambda\pm2K}\right.$$
$$\left.+\rho\kappa_1^{\lambda\pm2K}\right)D_{\rho\tau}^{\lambda\pm2Ki_1}=0, \tag{3.30}$$

$$\varepsilon_{q_1q_2}w_{q_1q_2}^{Ki_1}-\omega_{Ki_1}g_{q_1q_2}^{Ki_1}-G^{\lambda K}f^{\lambda K}(q_1q_2)v_{q_1q_2}^{(+)}D_{w\tau}^{\lambda Ki_1}=0. \tag{3.31}$$

Calculations of $E1$ transitions from the ground state to $I^\pi=1^-$ states with $K=0$ and 1 in even–even strongly deformed nuclei using dipole and octupole interactions were made in Ref. 19. The energies ω_{3Ki}, the $B(E3)$ values, and the largest components of the wave functions are mainly determined by the octupole interaction. The isovector dipole interaction has a weak influence on the energies, $B(E3)$, and structure of the states but has a large influence on $B(E1)$.

To describe magnetic $M\lambda$ transitions from the ground state to the one-phonon states, it is necessary to use the phonon operator (2.5) and take into account multipole *ph* and *pp* interactions and also magnetic spin–multipole *ph* interactions. The RPA equations for such cases have the form

$$\varepsilon_{q_1q_2}g_{q_1q_2}^{Ki_1}-\omega_{Ki_1}w_{q_1q_2}^{Ki_1}-f^{\lambda K}(q_1q_2)u_{q_1q_2}^{(+)}\sum_{\rho=\pm1}\left(\kappa_0^{\lambda K}\right.$$
$$\left.+\rho\kappa_1^{\lambda K}\right)D_{\rho\tau}^{\lambda Ki_1}-f^{\lambda K}(q_1q_2)v_{q_1q_2}^{(-)}G^{\lambda K}D_{g\tau}^{\lambda Ki_1}=0, \tag{3.32}$$

$$\varepsilon_{q_1q_2}w_{q_1q_2}^{Ki_1}-\omega_{Ki_1}g_{q_1q_2}^{Ki_1}-f^{\lambda K}(q_1q_2)v_{q_1q_2}^{(+)}G^{\lambda K}D_{w\tau}^{\lambda Ki_1}$$
$$-\sum_{\rho=\pm1}\left(\kappa_0^{L-1LK}\right.$$
$$\left.+\rho\kappa_1^{L-1LK}\right)D_{\rho\tau}^{L-1LKi_1}f^{L-1Lk}(q_1q_2)u_{q_1q_2}^{(-)}\chi(q_1q_2)=0. \tag{3.33}$$

From these equations we find $g_{q_1q_2}^{Ki_1}$ and $w_{q_1q_2}^{Ki_1}$, substitute them in the functions $D_\tau^{\lambda Ki_1}$, $D_{g\tau}^{\lambda Ki_1}$, $D_{w\tau}^{\lambda Ki_1}$, and $D_\tau^{L-1LKi_1}$, and obtain the system of equations

$$\sum_{\rho=\pm1}\left(\kappa_0^{\lambda K}+\rho\kappa_1^{\lambda K}\right)X_\tau^{\lambda Ki_1}D_{\rho\tau}^{\lambda Ki_1}-D_\tau^{\lambda Ki_1}+\sum_{\rho=\pm1}\left(\kappa_0^{L-1LK}\right.$$
$$\left.+\rho\kappa_1^{L-1LK}\right)Z_{2\tau}^{L-1LKi_1}D_{\rho\tau}^{L-1LKi_1}+G^{\lambda K}$$
$$\times\left[X_{\varepsilon\tau}^{\lambda Ki_1}D_{g\tau}^{\lambda Ki_1}+X_{\omega\tau}^{\lambda Ki_1}D_{w\tau}^{\lambda Ki_1}\right]=0, \tag{3.34}$$

$$\sum_{\rho=\pm1}\left[\left(\kappa_0^{\lambda K}+\rho\kappa_1^{\lambda K}\right)Z_\tau^{\lambda Ki_1}D_{\rho\tau}^{\lambda Ki_1}+\left(\kappa_0^{L-1LK}\right.\right.$$
$$\left.\left.+\rho\kappa_1^{L-1LK}\right)Z_\tau^{L-1LKi_1}D_{\rho\tau}^{L-1LKi_1}\right]-D_\tau^{L-1LKi_1}$$
$$+G^{\lambda K}\left[Z_{w\tau}^{\lambda LKi_1}D_{g\tau}^{\lambda Ki_1}+Z_{g\tau}^{\lambda LKi_1}D_{w\tau}^{\lambda Ki_1}\right]=0, \tag{3.35}$$

$$\sum_{\rho=\pm1}\left[\left(\kappa_0^{\lambda K}+\rho\kappa_1^{\lambda K}\right)X_{\varepsilon\tau}^{\lambda Ki_1}D_{\rho\tau}^{\lambda Ki_1}+\left(\kappa_0^{L-1LK}\right.\right.$$
$$\left.\left.+\rho\kappa_1^{L-1LK}\right)Z_{w\tau}^{\lambda LKi_1}D_{\rho\tau}^{L-1LKi_1}\right]+\left[G^{\lambda K}X_\tau^{\lambda Ki_1 -}-1\right]D_{g\tau}^{\lambda Ki_1}$$
$$+G^{\lambda K}X_{2\tau}^{\lambda Ki_1}D_{w\tau}^{\lambda Ki_1}=0, \tag{3.36}$$

$$\sum_{\rho=\pm1}\left[\left(\kappa_0^{\lambda K}+\rho\kappa_1^{\lambda K}\right)X_{\omega\tau}^{\lambda Ki_1}D_{\rho\tau}^{\lambda Ki_1}+\left(\kappa_0^{L-1LK}\right.\right.$$
$$\left.\left.+\rho\kappa_1^{L-1LK}\right)Z_{g\tau}^{\lambda LKi_1}D_{\rho\tau}^{L-1LKi_1}\right]+G^{\lambda K}X_{2\tau}^{\lambda Ki_1}D_{g\tau}^{\lambda Ki_1}$$
$$+\left[G^{\lambda K}X_\tau^{\lambda Ki_1 +}-1\right]D_{w\tau}^{\lambda Ki_1}=0. \tag{3.37}$$

Here the functions X are given by (3.14) and (3.15), in which 20 is replaced by λK and ω_{20i_1} by ω_{Ki_1};

$$Z_\tau^{L-1LKi_1}=\sum_{q_1q_2}{}^{\tau}\frac{[f^{L-1LK}(q_1q_2)u_{q_1q_2}^{(-)}]^2\varepsilon_{q_1q_2}}{\varepsilon_{q_1q_2}^2-\omega_{Ki_1}^2}, \tag{3.38}$$

$$Z_{\tau}^{\lambda LKi_1} = \sum_{q_1q_2} \tau \frac{f^{\lambda K}(q_1q_2)u^{(+)}_{q_1q_2} f^{L-1LK}(q_1q_2)u^{(-)}_{q_1q_2}\chi(q_1q_2)\omega_{Ki_1}}{\varepsilon^2_{q_1q_2}-\omega^2_{Ki_1}}, \tag{3.38'}$$

$$Z_{g\tau}^{\lambda LKi_1} = \sum_{q_1q_2} \tau \frac{f^{\lambda K}(q_1q_2)v^{(-)}_{q_1q_2} f^{L-1LK}(q_1q_2)u^{(-)}_{q_1q_2}\chi(q_1q_2)\varepsilon_{q_1q_2}}{\varepsilon^2_{q_1q_2}-\omega^2_{Ki_1}}, \tag{3.39}$$

$$Z_{w\tau}^{\lambda LKi_1} = \sum_{q_1q_2} \tau \frac{f^{\lambda K}(q_1q_2)v^{(+)}_{q_1q_2} f^{L-1LK}(q_1q_2)u^{(-)}_{q_1q_2}\chi(q_1q_2)\omega_{Ki_1}}{\varepsilon^2_{q_1q_2}-\omega^2_{Ki_1}}. \tag{3.39'}$$

The rank of the determinant of the system is 8.

We illustrate the unified description of the electric and magnetic interactions by a simple example. We consider a system consisting of one particle species (neutrons or protons) in order to obtain the explicit form of the functions $\psi^{Ki_1}_{q_1q_2}$ and $\phi^{Ki_1}_{q_1q_2}$. We restrict ourselves to *ph* interactions of electric and magnetic type. In this case the RPA equations (3.32) and (3.33) take the form

$$\varepsilon_{q_1q_2}g^{Ki_1}_{q_1q_2} - \omega_{Ki_1}w^{Ki_1}_{q_1q_2} - \kappa^{\lambda K}f^{\lambda K}(q_1q_2)u^{(+)}_{q_1q_2}D^{\lambda Ki_1}=0, \tag{3.40}$$

$$\varepsilon_{q_1q_2}w^{Ki_1}_{q_1q_2} - \omega_{Ki_1}g^{Ki_1}_{q_1q_2} - \kappa^{L-1LK}f^{L-1LK} \times (q_1q_2)u^{(-)}_{q_1q_2}\chi(q_1q_2)D^{L-1LKi_1}=0, \tag{3.41}$$

where $D^{\lambda Ki_1}$ and D^{L-1LKi_1} are given by (2.25) and (2.28). From these equations we find $g^{Ki_1}_{q_1q_2}$, $w^{Ki_1}_{q_1q_2}$ and substitute them in the functions $D^{\lambda Ki_1}$ and D^{L-1LKi_1}. As a result we obtain the secular equation

$$[\kappa^{\lambda K}X^{\lambda Ki_1}-1][\kappa^{L-1LK}Z^{L-1LKi_1}-1] = \kappa^{\lambda K}\kappa^{L-1LK}(Z^{\lambda LKi_1})^2,$$
$$D^{L-1LKi_1} = y_{\lambda LKi_1}D^{\lambda Ki_1}, \tag{3.42}$$

where $X^{\lambda Ki_1}$,Z^{L-1LKi_1}, and $Z^{\lambda LKi_1}$ are given by the expressions (3.14), (3.38), and (3.38');

$$y_{\lambda LKi_1} = \frac{\kappa^{\lambda K}Z^{\lambda LKi_1}}{1-\kappa^{L-1LK}Z^{L-1LKi_1}} = \frac{1-\kappa^{\lambda K}X^{\lambda Ki_1}}{\kappa^{L-1LK}Z^{\lambda LKi_1}}. \tag{3.43}$$

We substitute the functions $g^{Ki_1}_{q_1q_2}$ and $w^{Ki_1}_{q_1q_2}$ in the normalization condition (2.7) and obtain after factorization

$$\psi^{Ki_1}_{q_1q_2} = \frac{(2Y_{i_1})^{-1/2}}{\varepsilon_{q_1q_2}-\omega_{Ki_1}}[\kappa^{\lambda K}f^{\lambda K}(q_1q_2)u^{(+)}_{q_1q_2} + \kappa^{L-1LK}y_{\lambda LKi_1}f^{L-1LK}(q_1q_2)u^{(-)}_{q_1q_2}\chi(q_1q_2)], \tag{3.44}$$

$$\phi^{Ki_1}_{q_1q_2} = \frac{(2Y_{i_1})^{-1/2}}{\varepsilon_{q_1q_2}-\omega_{Ki_1}}[\kappa^{\lambda K}f^{\lambda K}(q_1q_2)u^{(+)}_{q_1q_2} - \kappa^{L-1LK}y_{\lambda LKi_1}f^{L-1LK}(q_1q_2)u^{(-)}_{q_1q_2}\chi(q_1q_2)], \tag{3.45}$$

where

$$Y_{i_1} = (\kappa^{\lambda K})^2Y_{\lambda i_1} + (\kappa^{L-1LK})^2Y_{Li_1} + \kappa^{\lambda K}\kappa^{L-1LK}Y_{\lambda LKi_1},$$

$$Y_{\lambda i_1} = \sum_{q_1q_2}\frac{(f^{\lambda K}(q_1q_2)u^{(+)}_{q_1q_2})^2\varepsilon_{q_1q_2}\omega_{Ki_1}}{(\varepsilon^2_{q_1q_2}-\omega^2_{Ki_1})^2},$$

$$Y_{Li_1} = \sum_{q_1q_2}\frac{(f^{LK}(q_1q_2)u^{(-)}_{q_1q_2})^2\varepsilon_{q_1q_2}\omega_{Ki_1}}{(\varepsilon^2_{q_1q_2}-\omega^2_{Ki_1})^2}, \tag{3.46}$$

$$Y_{\lambda LKi_1} = \sum_{q_1q_2} \times \frac{f^{\lambda K}(q_1q_2)u^{(+)}_{q_1q_2}f^{L-1LK}(q_1q_2)u^{(-)}_{q_1q_2}\chi(q_1q_2)(\varepsilon^2_{q_1q_2}+\omega^2_{Ki_1})}{(\varepsilon^2_{q_1q_2}-\omega^2_{Ki_1})^2}.$$

It can be seen from the expressions (3.44) and (3.45) for the amplitudes $\psi^{Ki_1}_{q_1q_2}$ and $\phi^{Ki_1}_{q_1q_2}$ that they consist of electric and magnetic parts.

Investigation of magnetic $M2$ and $M3$ transition probabilities in deformed nuclei showed[7] that the spin-multipole $\lambda' LK = L+1LK$ tensor interactions have a weak influence on the structure of states with excitation energies below 6 MeV and on the probabilities of $M2$ and $M3$ transitions. Therefore we do not take them into account in Eqs. (3.32) and (3.33). The energies and structure of the vibrational states below 6 MeV in even–even deformed nuclei are largely determined by multipole interactions. Inclusion of a spin-multipole magnetic interaction in addition to the multipole interactions leads to a shift of part of the $M2$ and $M3$ strength from the low-lying states to the region of the giant isovector resonance. The spin part of the $M2$ and $M3$ transitions is predominant, and the contribution of the orbital part to $B(M2)$ and $B(M3)$ is 10–40%.

We do not include in the QPNM Hamiltonian terms combining the operators

$$(Q^+_{Ki_1\sigma}Q^+_{Ki_2-\sigma} + Q_{Ki_1\sigma}Q_{Ki_2-\sigma}) \quad \text{and} \quad B(q_1q_2;K\sigma)B(q_1q_2;K-\sigma).$$

For the solutions of the RPA equation we must have fulfillment of the conditions

$$\left\langle Q_{Ki_1\sigma}\left\{\sum_{q\sigma}\varepsilon_q\alpha^+_{q\sigma}\alpha_{q\sigma} - \sum_{i_2i_3\sigma'}W^K_{i_2i_3}Q^+_{Ki_2\sigma'}Q_{Ki_3\sigma'}\right\}Q^+_{Ki_1\sigma}\right\rangle = \omega_{Ki_1} \tag{3.47}$$

and

$$\left\langle Q_{Ki_1\sigma}\left\{\sum_{q\sigma}\varepsilon_q\alpha^+_{q\sigma}\alpha_{q\sigma}-\sum_{i_2i_3}W^K_{i_2i_3}\frac{1}{2}\left[Q^+_{Ki_2\sigma}Q^+_{Ki_3-\sigma}\right.\right.\right.$$
$$\left.\left.\left.+Q_{Ki_3-\sigma}Q_{Ki_2\sigma}\right]\right\}Q^+_{Ki_1\sigma}\right\rangle=0. \qquad (3.48)$$

If we take into account the Pauli principle for the phonon operator, the condition (3.48) is not satisfied. The term of the Hamiltonian containing

$$(Q^+_{Ki_1\sigma}Q^+_{Ki_2-\sigma}+Q_{Ki_2-\sigma}Q_{Ki_1\sigma}),$$

is used in the multiphonon method[20] to describe two-phonon states in deformed nuclei.

The role of the terms of the Hamiltonian containing the operators

$$B(q_1q_2;K\sigma)B(q_1q_2;K-\sigma)$$

has been investigated. Numerical estimates in perturbation theory show that their influence on the vibrational states in strongly deformed nuclei is small. In nuclei of the transition region the influence of these terms is not small.

Calculations in the QPNM were made for nuclei with weak correlations in the ground states. The correlations in the ground states increase with increasing collectivity of the first one-phonon states. The particle–particle interactions reduce the correlations in the ground states and thus extend the region of applicability of the RPA. The mean number of quasiparticles with quantum number q in the ground state is

$$n_q=\sum_{\lambda\mu}n_q^{\lambda\mu}=\frac{1}{2}\sum_{\lambda\mu i}\sum_{q'}(\phi^{\lambda\mu i}_{qq'})^2. \qquad (3.49)$$

Earlier calculations of the mean number of quasiparticles in even–even deformed nuclei were made in Ref. 21. We calculated n_q and $n_q^{\lambda\mu}$ for ^{168}Er, ^{158}Gd, and ^{156}Gd. The greatest interest attaches to the values of n_q and $n_q^{\lambda\mu}$, $(n_q)_{\max}$, and $(n_q^{\lambda\mu})_{\max}$ that are maximal with respect to q. In accordance with the results for ^{168}Er in Ref. 14, $(n_q)_{\max}=0.017$ and $(n_q^{22})_{\max}=0.016$ for the proton state 411↓, $(n_q^{20})_{\max}=0.001$ for the neutron state 521↓, $(n_q^{30})_{\max}=0.0025$ for the proton state 400↑, and $(n_q^{31})_{\max}=0.0027$ for the neutron state 633↑. The correlations in the ground state for ^{168}Er are small. The gamma-vibrational state makes the largest contribution to them.

The numbers of quasiparticles in the ground state in ^{158}Gd are as follows: $(n_q)_{\max}=0.035$ and $(n_q^{20})_{\max}=0.02$ for the neutron state 521↑, $(n_q^{22})_{\max}=0.01$ and $(n_q^{30})_{\max}=0.002$ for the proton state 411↓, and $(n_q^{31})_{\max}=0.012$ for the proton state 532↑. The numbers of quasiparticles in the ground state of ^{156}Gd are as follows: $(n_q)_{\max}=0.04$ and $(n_q^{20})_{\max}=0.03$ for the neutron state 660↑, and $(n_q^{22})_{\max}=0.014$, $(n_q^{30})_{\max}=0.014$, and $(n_q^{31})_{\max}=0.024$ for the proton states 411↓, 411↑, and 532↑, respectively. The correlations in the ground state are stronger in ^{156}Gd and ^{158}Gd as compared with ^{168}Er. The largest contribution to them is made by the 0^+ states. This contribution is greater by a factor 30 in ^{156}Gd than it is in ^{168}Er. The $K^\pi=1^-$ state makes a relatively large contribution to the correlations in the ground state in both nuclei. The correlations in the ground state in ^{156}Gd are somewhat stronger than in ^{158}Gd. Nevertheless the correlations in the ground states in ^{156}Gd and ^{158}Gd are weak. A system of nonlinear equations that describes the correlations in the ground state outside the framework of the RPA was solved in Ref. 22. The contribution of the *pp* interaction was not taken into account in this paper. In this approximation the correlations in the ground state are stronger compared with the calculations of them using RPA phonons.

In nuclei lying on the boundaries of the region of deformed nuclei, especially in transition nuclei, the number of quasiparticles is increased to 0.3, and thus the RPA cannot be used. In strongly deformed nuclei the mean number of quasiparticles in the ground states is small, and the RPA is valid.

It may be concluded that if the *ph* and *pp* interactions are taken into account the RPA can be used to calculate states in strongly deformed nuclei in the regions $150<A<186$ $(90<N<112, 60<Z<86)$ and $A>232$. One-phonon states can be used to form the phonon basis in the QPNM.

4. EQUATIONS OF THE QPNM

4.1. The role of three-phonon terms

Our task is to describe in the framework of the QPNM the low-lying low-spin nonrotational states of strongly deformed even–even nuclei.

We take into account *ph* and *pp* multipole interactions. Usually our wave function contains one-phonon and two-phonon components. We have investigated the contribution of the two-phonon components to the wave functions of the low-lying states. To investigate these contributions, it is necessary to elucidate the role of the three-phonon terms. For this we write the wave function of an excited state in the form

$$\Psi_\nu(K_0^{\pi_0}\sigma_0)=\left\{\sum_{i_0}R^\nu_{i_0}Q^+_{g_0\sigma_0}+\sum_{\substack{g_1g_2\\ \sigma_1\sigma_2}}\frac{(1+\delta_{g_1g_2})^{1/2}}{2[1+\delta_{K_00}(1-\delta_{\mu_10})]^{1/2}}\right.$$
$$\times\delta_{\sigma_1\mu_1+\sigma_2\mu_2,\sigma_0K_0}P^\nu_{g_1g_2}Q^+_{g_1\sigma_1}Q^+_{g_2\sigma_2}$$
$$+\sum_{\substack{g_1g_2g_3\\ \sigma_1\sigma_2\sigma_3}}b_{g_1g_2g_3}\delta_{\sigma_1\mu_1+\sigma_2\mu_2+\sigma_3\mu_3,\sigma_0K_0}$$
$$\left.\times F^\nu_{g_1g_2g_3}Q^+_{g_1\sigma_1}Q^+_{g_2\sigma_2}Q^+_{g_3\sigma_3}\right\}\Psi_0. \qquad (4.1)$$

Here $g=\lambda\mu\, i$, $\mu_0=K_0$, and $b_{g_1g_2g_3}$ is a numerical factor that is given in Appendix 2; $\nu=1, 2, 3,...$ is the number of the $K_0^{\pi_0}$ state. To take into account the influence of the Pauli principle on the two- and three-phonon terms of the wave function (4.1), we introduce the function

$$\mathcal{K}^{K_0}(g_2,g_1'|g_1,g_2)=(1+\delta_{g_1g_2})^{-1}\sum_{\sigma_1\sigma_2}\delta_{\sigma_1\mu_1+\sigma_2\mu_2,\sigma_0K_0}\langle Q_{g_2\sigma_2}\times[[Q_{g_1'\sigma_1},Q^+_{g_1\sigma_1}],Q^+_{g_2\sigma_2}]\rangle, \quad (4.2)$$

$$\mathcal{K}^{K_0}(q_1g_2)\equiv\mathcal{K}^{K_0}(g_2,g_1|g_1,g_2). \quad (4.3)$$

Its explicit form is given in Appendix 2. The normalization condition for the wave function (4.1) in the diagonal approximation of the function $\mathcal{K}^{K_0}$ has the form

$$\sum_{i_0}(R^\nu_{i_0})^2+\sum_{g_1>g_2}(P^\nu_{g_1g_2})^2[1+\mathcal{K}^{K_0}(g_1g_2)]+\sum_{g_1>g_2>g_3}(F^\nu_{g_1g_2g_3})^2\left\{1+\frac{1}{2}[\mathcal{K}^{K_0\pm\mu_1}(g_2g_3)+\mathcal{K}^{K_0\pm\mu_2}(g_1g_3)+\mathcal{K}^{K_0\pm\mu_3}(g_1g_2)]\right\}=1. \quad (4.4)$$

We find the expectation value of

$$H^{\lambda_0K_0}_{\text{QPNM}}=\sum_{q\sigma}\varepsilon_q\alpha^+_{q\sigma}\alpha_{q\sigma}-\sum_{i_1i_2\sigma}W^{\lambda_0K_0}_{i_1i_2}Q^+_{\lambda_0K_0i_1\sigma}Q_{\lambda_0K_0i_2\sigma}+H^{\lambda_0K_0}_{vq}$$

for $n_{\max}=1$ with respect to the state (4.1):

$$(\Psi^*_\nu(K_0^{\pi_0}\sigma_0)H^{\lambda_0K_0}_{\text{QPNM}}\Psi_\nu(K_0^{\pi_0}\sigma_0))$$
$$=\sum_{i_0}\omega_{g_0}(R^\nu_{i_0})^2+\sum_{g_1>g_2}(P^\nu_{g_1g_2})^2\times[\omega_{g_1}+\omega_{g_2}+\Delta\omega(g_1,g_2)][1+\mathcal{K}^{K_0}(g_1g_2)]$$
$$+\sum_{g_1>g_2>g_3}(F^\nu_{g_1g_2g_3})^2[(\omega_{g_1}+\Delta\omega_{g_1})\times(1+\mathcal{K}^{K_0\pm\mu_1}(g_2g_3))+(\omega_{g_2}+\Delta\omega_{g_2})(1+\mathcal{K}^{K_0\pm\mu_2}(g_1,g_3))+(\omega_{g_3}+\Delta\omega_{g_3})(1+\mathcal{K}^{K_0\pm\mu_3}(g_1,g_2))]$$
$$-\sum_{i_0g_1g_2}\frac{(1+\delta_{g_1g_2})^{-1/2}}{[1+\delta_{K_00}(1-\delta_{\mu_10})]^{1/2}}R^\nu_{i_0}P^\nu_{g_1g_2}U^{g_0}_{g_1g_2}(1+\mathcal{K}^{K_0}(g_1g_2))$$
$$-\sum_{g_1g_2}\sum_{g_1'g_2'g_3'}\frac{(1+\delta_{g_1g_2})^{-1/2}}{[1+\delta_{K_00}(1-\delta_{\mu_10})]^{1/2}}\times b_{g_1'g_2'g_3'}P^\nu_{g_1g_2}F^\nu_{g_1'g_2'g_3'}U^{g_1g_2}_{g_1'g_2'g_3'}, \quad (4.5)$$

where

$$\Delta\omega(g_1g_2)=-\left[\frac{1-\delta_{K_00}}{1+\delta_{g_1g_2}}+\frac{\delta_{K_00}}{1+\delta_{g_1g_2}\delta_{\mu_10}}\right]\times\sum_{i'}\{\mathcal{K}^{K_0}(g_2,g_1'|g_1,g_2)W^{\lambda_1\mu_1}_{i_2i'}+\mathcal{K}^{K_0}(g_2',g_1|g_1,g_2)W^{\lambda_2\mu_2}_{i_1i'}\}. \quad (4.6)$$

Here $g_1'=\lambda_1\mu_1i'$, $g_2'=\lambda_2\mu_2i'$, and $W^{\lambda\mu}_{ii'}$ are determined by (2.14) and $H^{\lambda_0K_0}_{vq}$ by (2.29);

$$\Delta\omega_{g_1}=-\sum_{i'}W^{\lambda_1\mu_1}_{i_1i'}[\mathcal{K}^{k_0\pm\mu_3}(g_2,g_1'|g_1g_2)+\mathcal{K}^{K_0\pm\mu_2}(g_3,g_1'|g_1g_3)], \quad (4.7)$$

$$U^{g_0}_{g_1g_2}(1+\mathcal{K}^{K_0}(g_1,g_2))=-\tfrac{1}{2}\sum_{\sigma_1\sigma_2}\delta_{\sigma_1\mu_1+\sigma_2\mu_2,\sigma_0K_0}\times[\langle Q_{g_0\sigma_0}\widetilde{H}^{K_0}_{vq}Q^+_{g_1\sigma_1}Q^+_{g_2\sigma_2}\rangle+\text{h.c.}], \quad (4.8)$$

$$U^{g_1g_2}_{g_1'g_2'g_3'}=\delta_{g_1g_1'}U^{g_2}_{g_2'g_3'}(1+\mathcal{K}^{K_2}(g_2'g_3'))+\delta_{g_1g_2'}U^{g_2}_{g_1'g_3'}(1+\mathcal{K}^{K_2}(g_1'g_3'))+\ldots. \quad (4.9)$$

The explicit form of the functions $U^{g_1}_{g_2g_3}$ is given in Appendix 2. Using the variational principle in the form

$$\delta\{(\Psi_\nu'(K_0^{\pi_0}\sigma_0)H^{\lambda_0K_0}_{\text{QPNM}}\Psi_\nu(K_0^{\pi_0}\sigma_0))-E_\nu[(\Psi^*_\nu(K_0^{\pi_0}\sigma_0)\Psi_\nu(K_0^{\pi_0}\sigma_0))-1]\}=0, \quad (4.10)$$

we obtain equations for the energies E_ν and the wave functions (4.1); they have the form

$$(\omega_{g_0}-E_\nu)R^\nu_{i_0}-\sum_{g_1>g_2}\frac{(1+\delta_{g_1g_2})^{-1/2}}{[1+\delta_{K_0,0}(1-\delta_{\mu_10})]^{1/2}}P^\nu_{g_1g_2}U^{g_0}_{g_1g_2}\times(1+\mathcal{K}^{K_0}(g_1,g_2))=0, \quad (4.11)$$

$$[\omega_{g_1}+\omega_{g_2}+\Delta\omega(g_1g_2)-E_\nu](1+\mathcal{K}^{K_0}(g_1,g_2))P^\nu_{g_1g_2}-\sum_{i_0}\frac{(1+\delta_{g_1g_2})^{-1/2}}{[1+\delta_{K_0,0}(1-\delta_{\mu_10})]^{1/2}}U^{g_0}_{g_1g_2}\times(1+\mathcal{K}^{K_0}(g_1,g_2))R^\nu_{i_0}-\frac{(1+\delta_{g_1g_2})^{-1/2}}{[1+\delta_{K_0,0}(1-\delta_{\mu_10})]^{1/2}}\times\sum_{g_1'g_2'g_3'}b_{g_1'g_2'g_3'}U^{g_1g_2}_{g_1'g_2'g_3'}F^\nu_{g_1'g_2'g_3'}=0, \quad (4.12)$$

$$\left\{(\omega_{g_1'}+\Delta\omega_{g_1'})\left[1+\mathcal{K}^{K_0\pm\mu_1'}(g_2'g_3')\right]+(\omega_{g_2'}+\Delta\omega_{g_2'})\times\left[1+\mathcal{K}^{K_0\pm\mu_2'}(g_1'g_3')\right]+(\omega_{g_3'}+\Delta\omega_{g_3'})\times\left[1+\mathcal{K}^{K_0\pm\mu_3'}(g_1'g_2')\right]-E_\nu\left[1+\mathcal{K}^{K_0\pm\mu_1'}(g_2'g_3')+\mathcal{K}^{K_0\pm\mu_2'}(g_1'g_3')+\mathcal{K}^{K_0\pm\mu_3'}(g_1'g_2')\right]\right\}F^\nu_{g_1'g_2'g_3'}$$
$$-\sum_{g_1g_2}\frac{(1+\delta_{g_1g_2})^{-1/2}}{[1+\delta_{K_0,0}(1-\delta_{\mu_10})]^{1/2}}b_{g_1'g_2'g_3'}U^{g_1g_2}_{g_1'g_2'g_3'}P^\nu_{g_1g_2}=0. \quad (4.13)$$

If we find $F^{\nu}_{g_1g_2g_3}$ from (4.13) and $R^{\nu}_{i_0}$ from (4.11) and substitute in Eq. (4.12), we obtain

$$\sum_{g_1>g_2} P^{\nu}_{g_1g_2}\Bigg\{[\omega_{g_1}+\omega_{g_2}+\Delta\omega(g_1g_2)-E_\nu](1+\mathscr{K}^{K_0}(g_1,g_2))\delta_{g_1,g_1^0}\delta_{g_2,g_2^0}$$
$$-\sum_{i_0}\frac{(1+\delta_{g_1g_2})^{-1/2}}{[1+\delta_{K_0,0}(1-\delta_{\mu_10})]^{1/2}}\frac{(1+\delta_{g_1^0g_2^0})^{-1/2}}{[1+\delta_{K_0,0}(1-\delta_{\mu_1^00})]^{1/2}}\frac{U^{g_0}_{g_1^0g_2^0}U^{g_0}_{g_1g_2}(1+\mathscr{K}^{K_0}(g_1^0,g_2^0))(1+\mathscr{K}^{K_0}(g_1,g_2))}{\omega_{g_0}-E_\nu}$$
$$-\sum_{g_1'g_2'g_3'}\frac{(1+\delta_{g_1g_2})^{-1/2}}{[1+\delta_{K_0,0}(1-\delta_{\mu_10})]^{1/2}}\frac{(1+\delta_{g_1^0g_2^0})^{-1/2}}{[1+\delta_{K_0,0}(1-\delta_{\mu_1^00})]^{1/2}}$$
$$\times b^2_{g_1'g_2'g_3'}\frac{U^{g_1^0g_2^0}_{g_1'g_2'g_3'}U^{g_1g_2}_{g_1'g_2'g_3'}}{J(g_1',g_2',g_3')-E_\nu[1+\mathscr{K}^{K_0\pm\mu_1'}(g_2',g_3')+\mathscr{K}^{K_0\pm\mu_2'}(g_1',g_3')+\mathscr{K}^{K_0\pm\mu_3'}(g_1',g_2')]}\Bigg\}=0, \quad (4.14)$$

where

$$J(g_1',g_2',g_3')=(\omega_{g_1'}+\Delta\omega_{g_1'})(1+\mathscr{K}^{K_0\pm\mu_1'}(g_2',g_3'))$$
$$+(\omega_{g_2'}+\Delta\omega_{g_2'})(1+\mathscr{K}^{K_0\pm\mu_2'}(g_1',g_3'))$$
$$+(\omega_{g_3'}+\Delta\omega_{g_3'})(1+\mathscr{K}^{K_0\pm\mu_3'}(g_1',g_2')).$$

To describe the fragmentation of a two-phonon state, it is necessary to find a solution of the system of equations (4.11–4.13) with allowance for the normalization condition (4.4). In this case we are not justified in ignoring the nondiagonal terms in Eq. (4.12), since otherwise spurious solutions appear.[23] We investigated the influence of the three-phonon terms in the wave function (4.1) on the contribution of the two-phonon configurations to the wave functions of low-lying states. In this case we can ignore the nondiagonal terms in Eq. (4.14) and make a restriction to the terms with $g_1=g_1^0$ and $g_2=g_2^0$. In such an approximation the influence of the three-phonon terms reduces to a shift of the two-phonon poles, which we denote as $\Delta(g_1g_2)$. This shift is nonvanishing if for the phonon operators in the three-phonon terms of the wave function (4.1) bosonic commutational relations are applied. The contribution of $\Delta(g_1g_2)$ is in the opposite sense to the contribution of the shift $\Delta\omega(g_1g_2)$, and therefore allowance for the three-phonon terms reduces the value of the two-phonon pole.

In such a diagonal approximation we obtain two equations, which are (4.11) and

$$[\omega_{g_1}+\omega_{g_2}+\Delta\omega(g_1g_2)+\Delta(g_1g_2)-E_\nu]P^{\nu}_{g_1g_2}$$
$$-\sum_{i_0}\frac{(1+\delta_{g_1g_2})^{-1/2}}{[1+\delta_{K_0,0}(1-\delta_{\mu_10})]^{1/2}}U^{g_0}_{g_1g_2}R^{\nu}_{i_0}=0. \quad (4.15)$$

From this we obtain a secular equation in the form

$$\det\left\|(\omega_{g_0}-E_\nu)\delta_{i_0,i_0'}-\sum_{g_1>g_2}\frac{(1+\mathscr{K}^{K_0}(g_1g_2)}{(1+\delta_{g_1g_2})[1+\delta_{K_0,0}(1-\delta_{\mu_10})]}\times\frac{U^{g_0}_{g_1g_2}U^{g_0'}_{g_1g_2}}{\omega_{g_1}+\omega_{g_2}+\Delta\omega(g_1g_2)+\Delta(g_1g_2)-E_\nu}\right\|=0. \quad (4.16)$$

This secular equation differs from the one used in Refs. 3, 16, and 17 by the additional shift $\Delta(g_1g_2)$.

The order of the determinant (4.16) is equal to the number of one-phonon terms in the wave function (4.1). Inclusion of the Pauli principle in the two-phonon terms (4.1) leads to the occurrence in (4.16) of the factor $(1+\mathscr{K}^{K_0}(g_1g_2))$ and the shift $\Delta\omega(g_1g_2)$ of the two-phonon poles. The three-phonon terms in (4.1) give the additional shift $\Delta(g_1g_2)$.

The form of Eqs. (4.11), (4.12), and (4.13) and the order of the corresponding determinant do not depend on which *ph* and *pp* multipole and spin–multipole interactions are taken into account and are also independent of the rank $n_{\max}$ of the separable interactions. Equations (4.11), (4.15), and (4.16) are the same as the equations of the previous studies of Refs. 1 and 3, in which only *ph* multipole interactions were taken into account. All the complications that arise from the form of the interactions are manifested in the RPA equations. This means that in the framework of the QPNM one can use arbitrary complicated interactions represented in separable form.

Many-phonon terms have been taken into account in the wave functions of excited states in a number of studies, for example, in Refs. 20, 24, and 25.

4.2. Equations of the QPNM with wave function containing one- and two-phonon terms

Calculations of the energies and wave functions of nonrotational states of even–even strongly deformed nuclei are often made with a wave function containing one- and two-phonon terms in the form

$$\Psi_\nu(K_0^{\pi_0}\sigma_0)=\Bigg\{\sum_{i_0} R^\nu_{i_0} Q^+_{\lambda_0 K_0 i_0\sigma_0} + \sum_{\substack{\lambda_1 K_1 i_1\sigma_1\\ \lambda_2 K_2 i_2\sigma_2}} \frac{(1+\delta_{\lambda_1\lambda_2}\delta_{K_1K_2}\delta_{i_1i_2})^{1/2}}{2[1+\delta_{K_0,0}(1-\delta_{K_1,0})]^{1/2}} \times\delta_{\sigma_1K_1+\sigma_2K_2,\sigma_0K_0} P^\nu_{\lambda_1K_1i_1\lambda_2K_2i_2} \times Q^+_{\lambda_1K_1i_1\sigma_1}Q^+_{\lambda_2K_2i_2\sigma_2}\Bigg\}\Psi_0. \quad (4.17)$$

Its normalization condition has the form

$$\sum_{i_0}(R^\nu_{i_0})^2+\sum_{(\lambda_1K_1i_1)>(\lambda_2K_2i_2)}(P_{\lambda_1K_1i_1,\lambda_2K_2i_2})^2 \times[1+\mathcal{K}^{K_0}(\lambda_1K_1i_1,\lambda_2K_2i_2)]=1. \quad (4.18)$$

The equations for finding the energies E_ν and functions $R^\nu_{i_0}$ and P^ν have the form

$$(\omega_{\lambda_0K_0i_0}-E_\nu)R^\nu_{i_0}-\sum_{\lambda_1K_1i_1>\lambda_2K_2i_2}\frac{(1+\delta_{\lambda_1\lambda_2}\delta_{K_1K_2}\delta_{i_1i_2})^{-1/2}}{[1+\delta_{K_0,0}(1-\delta_{K_1,0})]^{1/2}} \times P^\nu_{\lambda_1K_1i_1,\lambda_2K_2i_2}U^{\lambda_0K_0i_0}_{\lambda_1K_1i_1,\lambda_2K_2i_2} \times[1+\mathcal{K}^{K_0}(\lambda_1K_1i_1,\lambda_2K_2i_2)]=0. \quad (4.19)$$

$$[\omega_{\lambda_1K_1i_1}+\omega_{\lambda_2K_2i_2}+\Delta\omega(\lambda_1K_1i_1,\lambda_2K_2i_2) \times+\Delta(\lambda_1K_1i_1,\lambda_2K_2i_2)-E_\nu]P^\nu_{\lambda_1K_1i_1,\lambda_2K_2i_2} -\frac{(1+\delta_{\lambda_1\lambda_2}\delta_{K_1K_2}\delta_{i_1i_2})^{-1/2}}{[1+\delta_{K_0,0}(1-\delta_{K_10})]^{1/2}}\sum_{i_0}R^\nu_{i_0}U^{\lambda_0K_0i_0}_{\lambda_1K_1i_1,\lambda_2K_2i_2}=0. \quad (4.20)$$

The determinant of this system has the form (4.15), and its order is equal to the number of one-phonon terms in the wave function (4.17). The shift $\Delta(\lambda_1K_1i_1,\lambda_2K_2i_2)$ is due to the allowance in the diagonal approximation for the contribution from the three-phonon terms added to the wave function (4.17). It has sign opposite to the sign of the shift $\Delta\omega(\lambda_1K_1i_1,\lambda_2K_2i_2)$, i.e., it reduces the magnitude of the two-phonon pole. In accordance with a numerical estimate it is approximately $0.2\Delta\omega(\lambda_1K_1i_1,\lambda_2K_2i_2)$.

The function $U^{\lambda_0K_0i_0}_{\lambda_1K_1i_1,\lambda_2K_2i_2}$ is an incoherent sum of many terms that contain the matrix elements $f^{\lambda K}(q_1q_2)$ and the direct and inverse RPA amplitudes $\psi^{Ki}_{q_1q_2}$ and $\phi^{Ki}_{q_1q_2}$, respectively. In deformed nuclei the terms with opposite signs cancel each other. As a result the numerical values of this function range from 0.01 to 0.20 MeV and only in some cases take values greater than 0.2 MeV. In spherical nuclei the largest terms in $U^{\lambda_0i_0}_{\lambda_1i_1,\lambda_2i_2}$ have the same sign for the first roots of the secular equation, and therefore the numerical values of $U^{\lambda_0i_0}_{\lambda_1i_1,\lambda_2i_2}$ in a spherical nucleus with open shells are greater by one or two orders of magnitude than in deformed nuclei.

In order to investigate the influence of a density-dependent interaction on the function $U^{\lambda_0K_0i_0}_{\lambda_1K_1i_1,\lambda_2K_2i_2}$, we estimated its values using RPA wave functions calculated with the interaction $W^{KE}_{i_1,i_2}$ in the form represented in (3.27). We took into account density-independent separable forces, and also forces having a maximum on the surface of the nucleus. According to our calculations, the numerical value of $U^{441}_{221,221}$ in ^{168}Er is increased by 10% for $\zeta=0.1$ as compared with $\zeta=0$. This means that a density-dependent separable interaction has a weak influence on the low-lying vibrational states in strongly deformed nuclei.

The terms of the QPNM Hamiltonian that contain the operators

$$(Q^+_{Ki_1\sigma}Q^+_{Ki_2-\sigma}+Q_{Ki_2-\sigma}Q_{Ki_1\sigma}),$$

do not change the poles of the secular equation (4.15) but act on the function $U^{\lambda_0K_0i_0}_{\lambda_1K_1i_1,\lambda_2K_2i_2}$. According to our estimates, this influence is small.

4.3. $E\lambda$ and $M\lambda$ transitions from the ground state to excited states and between excited states

We write down expressions for the reduced probabilities of $E\lambda$ transitions from the ground state to an excited state and of $E\lambda$ and $M\lambda$ transitions between excited states. The reduced probabilities of $E\lambda$ and $M\lambda$ transitions from the $0^+_{\rm g.s.}$ ground state to an excited state with fixed values of $I^\pi K_\nu$ can be expressed as

$$B(E\lambda;0^+_{\rm g.s.}\to I^\pi K_\nu) =(2-\delta_{\mu0})\langle 00\lambda\mu|IK\rangle^2e^2 \times\left[\frac{1+\delta_{\mu0}}{2}\sum_i R^\nu_i\sum_\tau e^{(\lambda)}_{\rm eff}(\tau)\sum_{qq'}{}^\tau p^{\lambda\mu}(qq')u^{(+)}_{qq'}g^{\lambda\mu i}_{qq'}\right]^2, \quad (4.21)$$

$$B(M\lambda;0^+_{\rm g.s.}\to I^\pi K_\nu) =(2-\delta_{\mu0})\langle 00\lambda\mu|IK\rangle^2\mu_N^2 \times\left[\frac{1}{2}\sum_i R^\nu_i\sum_\tau\sum_{qq'}{}^\tau\Gamma_\tau(M\lambda\mu;qq')u^{(-)}_{qq'}w^{\lambda\mu i}_{qq'}\chi(qq')\right]^2, \quad (4.22)$$

where $p^{\lambda\mu}(qq')$ is the single-particle matrix element $f^{\lambda\mu}(qq')$ with radial dependence r^λ instead of $\partial V/\partial r$; $e^{(\lambda)}_{\rm eff}(\tau)$ is the proton or neutron effective charge;

$$\Gamma_\tau(M\lambda\mu;qq') = \sqrt{\lambda(2\lambda+1)}\left\langle q\left| r^{\lambda-1}\left[\frac{1}{2}g_s^{\rm eff}(\tau)(\sigma\mathbf{Y}_{\lambda-1})_{\lambda\mu} + g_l^{\rm eff}(\tau)\frac{2}{\lambda+1}(1\mathbf{Y}_{\lambda-1})_{\lambda\mu}\right]\right| q'\right\rangle, \qquad (4.23)$$

$$\Gamma_\tau(M1\mu;qq') = \sqrt{\frac{3}{4\pi}}\left\langle q\left|\frac{1}{2}g_s^{\rm eff}(\tau)\sigma + g_l^{\rm eff}(\tau)1\right|q'\right\rangle, \qquad (4.24)$$

$g_l^{\rm eff}(p)=1$, $g_l^{\rm eff}(n)=0$, $g_s^{\rm eff}(\tau)$ is the effective spin gyromagnetic ratio, $g_s^{\rm eff}(\tau)=g^{\rm eff}g_s^{\rm (free)}(\tau)$, $1/2\sigma$ is the spin, and μ_N is the nuclear magneton. The reduced probabilities of $E\lambda$ or $M\lambda$ transitions from the initial state $I_0\lambda_0K_0^{\pi_0}\nu_0$ to the final state $I_f\lambda_fK_f^{\pi_f}\nu_f$ have the form

$$B(\lambda\mu;I_0^{\pi_0}K_0\nu_0\to I_f^{\pi_f}K_f\nu_f) = [1+(\delta_{K_0 0}-\delta_{K_f 0})^2]\{\langle I_0K_0\lambda K_f-K_0|I_fK_f\rangle^2|(\Psi^*_{\nu_f}(K_f^{\pi_f}\sigma_f=\sigma_0)\mathfrak{M}(\lambda;\mu=|K_f-K_0|)\Psi_{\nu_0}(K_0^{\pi_0}\sigma_0))|^2 + \langle I_0-K_0\lambda K_f+K_0|I_fK_f\rangle^2|(\Psi^*_{\nu_f}(K_f^{\pi_f}\sigma_f=-\sigma_0)\mathfrak{M}(\lambda;\mu=K_0+K_f)\Psi_{\nu_0}(K_0^{\pi_0}\sigma_0))|^2\}, \qquad (4.25)$$

$$(\Psi^*_{\nu_f}(K_f^{\pi_f}\sigma_f)\mathfrak{M}(\lambda;\mu=|K_f\pm K_0)\Psi_{\nu_0}(K_0^{\pi_0}\sigma_0)) = \sum_{i_0 i_f} R_{i_0}^{\nu_0}R_{i_f}^{\nu_f}\mathfrak{L}_{i_0 i_f}^{\lambda\mu} + \sum_{i_3} M_{i_3}^{\lambda\mu}T_{i_3}^{\lambda\mu}. \qquad (4.26)$$

Here

$$\mathfrak{L}_{i_0 i_f}^{\lambda\mu}(E) = \sum_\tau e_{\rm eff}^{(\lambda)}(\tau)\sum_{q_1q_2q_3} {}^\tau p^{\lambda\mu}(q_1q_2)v_{q_1q_2}^{(-)} \times\left[\psi_{q_2q_3}^{\lambda_0K_0i_0}\psi_{q_1q_3}^{\lambda_fK_fi_f} + \phi_{q_1q_3}^{\lambda_0K_0i_0}\phi_{q_2q_3}^{\lambda_fK_fi_f}\right], \qquad (4.27)$$

$$\mathfrak{L}_{i_0 i_f}^{\lambda\mu}(M) = \sum_\tau \sum_{q_1q_2q_3} {}^\tau\Gamma_\tau(M\lambda\mu;q_1q_2)v_{q_1q_2}^{(+)} \times\left[\psi_{q_2q_3}^{\lambda_0K_0i_0}\psi_{q_1q_3}^{\lambda_fK_fi_f} - \phi_{q_1q_3}^{\lambda_0K_0i_0}\phi_{q_2q_3}^{\lambda_fK_fi_f}\right], \qquad (4.28)$$

$$M_{i_3}^{\lambda\mu}(E) = \frac{1}{2}\sum_\tau e_{\rm eff}^{(\lambda)}(\tau)\sum_{qq'} {}^\tau p^{\lambda\mu}(qq')u_{qq'}^{(+)}g_{qq'}^{\lambda_3\mu i_3}, \qquad (4.29)$$

$$M_{i_3}^{\lambda\mu}(M) = \frac{1}{2}\sum_\tau\sum_{qq'} {}^\tau\Gamma_\tau(M\lambda\mu;qq')u_{qq'}^{(-)}\chi(qq')w_{qq'}^{\lambda_3\mu i_3}, \qquad (4.30)$$

$$T_{i_3}^{\lambda\mu} = \sum_{i_0} R_{i_0}^{\nu_0}P_{g_0g_3}^{\nu_f}[1+\mathcal{K}^{K_f}(g_0g_3)](1+\delta_{g_0g_3})^{1/2} \times[1+\delta_{K_f,0}(1-\delta_{K_0,0}]^{-1/2} \pm \sum_{i_f} R_{i_f}^{\nu_f}P_{g_fg_3}^{\nu_0}[1+\mathcal{K}^{K_0}(g_fg_3)](1+\delta_{g_fg_3})^{1/2} [1+\delta_{K_0,0}(1-\delta_{K_f,0})]^{-1/2}, \qquad (4.31)$$

where in (4.31) the signs + and − are used for electric and magnetic transitions, respectively, $g_0=\lambda_0K_0i_0$, $g_f=\lambda_fK_fi_f$, $g_3=\lambda_3ui_3$. We ignore the probabilities of gamma transitions between the two-phonon terms of the wave functions of the initial and final states because they are small. We take into account transitions between the one- and two-phonon terms. To calculate the matrix elements of gamma transitions between excited states, we make the following transformations. We express the operator $(A^+(qq',\mu\sigma)+A(qq';\mu-\sigma))$ in terms of the phonon operator $Q^+_{\lambda\mu i}$ in the operators of the gamma transitions. For the $E1$ transition octupole phonons with $g_3=\lambda_3\mu i_3$, $\lambda_3=3$, $\mu=0$, and 1 are used and for the $E2$ transition quadrupole phonons with $\lambda_3=2$, $\mu=0$, 1, and 2. In the operators of the $M1$ transition quadrupole phonons with $g_3=\lambda_3\mu i_3$, $\lambda_3=2$, and $\mu=0$ and 1 are used and in the $M2$ transition octupole phonons with $\lambda_3=3$, $\mu=1$ and 2. The low-lying $K^\pi=0^\pm$, $1^\pm$, and $2^\pm$ states are treated as quadrupole and octupole vibrational states. As is shown in Ref. 19, the isovector dipole interaction leads to a renormalization of the constant of the isoscalar octupole interaction and to a decrease in the $B(E1;0^+_{g.s.}\to 1^-K)$ values. This has a weak effect on the largest components of the wave functions of the $K^\pi=0^-$ and 1^- one-phonon states. The spin-multipole $\lambda\lambda-1K$ interaction has a weak effect on the wave functions of quadrupole and octupole states.[7] Therefore in our calculations of the gamma transition between the one- and two-phonon terms of the wave functions of the initial and final states we do not take into account the isovector dipole interaction and the spin-multipole interaction.

There are experimental data on the intensities of gamma transitions between excited states. We compare the experimental ratio of the intensities with the results of the calculations. We use the following expressions for the probability of decay per second:[26]

$$W(E1)=1.59\cdot 10^{15}E_\gamma^3B(E1), \qquad (4.32)$$

$$W(E2)=1.22\cdot 10^{9}E_\gamma^5B(E2), \qquad (4.33)$$

$$W(E3)=5.67\cdot 10^{2}E_\gamma^7B(E3), \qquad (4.34)$$

$$W(M1)=1.76\cdot 10^{13}E_\gamma^3B(M1), \qquad (4.35)$$

$$W(M2)=1.35\cdot 10^{7}E_\gamma^5B(M2), \qquad (4.36)$$

where the photon energy E_γ is given in mega-electron-volts, $B(E\lambda)$ is measured in units of $e^2fm^{2\lambda}$, and $B(M\lambda)$ in units of $\mu_Nfm^{2\lambda-2}$.

5. CONCLUSIONS

In this review we have presented the mathematical formalism of the QPNM constructed to describe the energies and wave functions of low-lying states of even–even strongly deformed nuclei. This formalism is suitable for describing excited states of the nuclei for which the correlations in the ground states are not strong. In the framework of the QPNM one can calculate the probabilities of electric $E\lambda$ and magnetic $M\lambda$ transitions from the ground state to excited states and between excited states. One can compare the largest two-quasiparticle components of the wave functions as extracted from experimental data on single-nucleon transfer reactions and unhindered allowed β decays with the two-quasiparticle contributions to the dominant one-phonon component. For 0^+ states one can calculate the values of ρ^2 associated with $E0$ transitions, $X(E0/E2)$, which are determined by the reduced probabilities of $E0$ and $E2$ transitions, and also the spectroscopic factors of (t,p) and (p,t) reactions normalized by those for transitions between the ground states. The Coriolis interaction with the calculated wave functions of nonrotational states can be calculated usually in the cases when its influence is not large.

The mathematical formalism of the QPNM can be used to describe excited states of even–even strongly deformed nuclei. It provides a basis for further study of the properties of deformed nuclei.

APPENDIX 1

We give the operators, matrix elements, and functions that are encountered in the QPNM Hamiltonian:

$$A^{(+)}(q_1q_2;K\sigma) = \begin{cases} \widetilde{A}^+(q_1q_2;K\sigma) = \sum_{\sigma'} \delta_{\sigma'(K_1-K_2),\sigma K}\sigma' \alpha^+_{q_1\sigma'}\alpha^+_{q_2-\sigma'}, & \text{if } |K_1-K_2|=K, \\ \bar{A}^+(q_1q_2;K\sigma) = \sum_{\sigma'} \delta_{\sigma'(K_1+K_2),\sigma K}\alpha^+_{q_2\sigma'}\alpha^+_{q_1\sigma'}, & \text{if } |K_1+K_2|=K; \end{cases} \tag{A1}$$

$$a^{(+)}(q_1q_2;K\sigma) = \begin{cases} \widetilde{a}^+(q_1q_2;K\sigma) = \sum_{\sigma'} \delta_{\sigma'(K_1-K_2),\sigma K}\alpha^+_{q_1\sigma'}\alpha^+_{q_2-\sigma'}, & \text{if } |K_1-K_2|=K, \\ \bar{a}^+(q_1q_2;K\sigma) = \sum_{\sigma'} \delta_{\sigma'(K_1+K_2),\sigma K}\sigma'\alpha^+_{q_2\sigma'}\alpha^+_{q_1\sigma'}, & \text{if } |K_1+K_2|=K; \end{cases} \tag{A2}$$

$$\widetilde{a}^+(q_1q_2;K\sigma) = \sigma\chi(q_1q_2)\widetilde{A}^+(q_1q_2;K\sigma),$$

$$\bar{a}^+(q_1q_2;K\sigma) = \sigma\bar{A}^+(q_1q_2;K\sigma). \tag{A3}$$

$$\chi(q_1q_2) = -\chi(q_2q_1), \quad \chi^2(q_1q_2) = 1,$$

$$\chi(q_2q_1)a^+(q_1q_2;K\sigma) = -a^+(q_1q_2;K\sigma) = a^+(q_2q_1;K\sigma);$$

$$\delta_{\sigma_1K_1+\sigma_2K_2,\sigma K} = \begin{cases} 1, & \text{if } \sigma_1K_1+\sigma_2K_2=\sigma K, \\ 0, & \text{if } \sigma_1K_1+\sigma_2K_2\neq\sigma K, \end{cases}$$

for all $K>0$, $\sigma=\pm1$. After simple manipulations we express the operators $\widetilde{A}(q_1q_2;K\sigma)$, $\bar{A}(q_1q_2;K\sigma)$, and $a(q_1q_2;K\sigma)$ in terms of the phonon operator (2.5):

$$\widetilde{A}^+(q_1q_2;K\sigma) = \frac{1-i\sigma}{\sqrt{2}} \sum_{i_0} \left[\psi^{Ki_0}_{q_1q_2}Q^+_{Ki_0\sigma} + \phi^{Ki_0}_{q_1q_2}Q^+_{Ki_0-\sigma}\right],$$

$$\bar{A}^+(q_1q_2;K\sigma) = \frac{1-i\sigma}{\sqrt{2}}\chi(q_1q_2) \sum_{i_0} \left[\psi^{Ki_0}_{q_1q_2}Q^+_{Ki_0\sigma} + \phi^{Ki_0}_{q_1q_2}Q^+_{Ki_0-\sigma}\right], \tag{A4}$$

$$a^+(q_1q_2;K\sigma) = \frac{\sigma-i}{\sqrt{2}}\chi(q_1q_2) \sum_{i_0} \left[\psi^{Ki_0}_{q_1q_2}Q^+_{Ki_0\sigma} + \phi^{Ki_0}_{q_1q_2}Q^+_{Ki_0-\sigma}\right].$$

$$B(q_1q_2;K\sigma) = \begin{cases} \sum_{\sigma'} \delta_{\sigma'(K_1-K_2),\sigma K}\alpha^+_{q_1\sigma'}\alpha_{q_2\sigma'}, & \text{if } |K_1-K_2|=K, \\ \sum_{\sigma'} \delta_{\sigma'(K_1+K_2),\sigma K}\sigma'\alpha^+_{q_1\sigma'}\alpha_{q_2-\sigma'}, & \text{if } |K_1+K_2|=K. \end{cases} \tag{A5}$$

$$B(q_1q_2;K=0) = \sum_{\sigma} \alpha^+_{q_1\sigma}\alpha_{q_2\sigma},$$

$$\mathfrak{B}(q_1q_2;K\sigma) = \begin{cases} \sum_{\sigma'} \delta_{\sigma'(K_1-K_2),\sigma K}\sigma'\alpha^+_{q_1\sigma'}\alpha_{q_2\sigma'}, & \text{if } |K_1-K_2|=K, \\ \sum_{\sigma'} \delta_{\sigma'(K_1+K_2),\sigma K}\alpha^+_{q_1\sigma'}\alpha_{q_2-\sigma'}, & \text{if } |K_1+K_2|=K. \end{cases} \tag{A6}$$

The matrix elements of the multipole and spin–multipole operators can be expressed as follows:

$$f^{\lambda K}_n(q_1q_2) = \langle q_1|R^\lambda_n(r)Y_{\lambda K}(\theta\phi)|q_2\rangle = \begin{cases} \widetilde{f}^{\lambda K}_n(q_1q_2), & \text{if } |K_1-K_2|=K; \\ \bar{f}^{\lambda K}_n(q_1q_2)\chi(q_1q_2), & \text{if } |K_1+K_2|=K; \end{cases} \tag{A7}$$

$$f^{L\pm1LK}_n(q_1q_2) = \langle q_1|R^{L\pm1}_n(r)\{\sigma Y_{L\pm1}(\theta\phi)\}_{LK}|q_2\rangle,$$

$$f^{\lambda\lambda K}_n(q_1q_2) = \widetilde{f}^{\lambda\lambda K}_n(q_1q_2)\chi(q_1q_2) + \bar{f}^{\lambda\lambda K}(q_1q_2). \tag{A8}$$

For simple separable interactions with radial dependence $R(r)=\partial V(r)/\partial r$, where $V(r)$ is the central part of the Woods–Saxon potential, the matrix elements have the form

$$f^{\lambda K}(q_1q_2) = \left\langle q_1\left|\frac{\partial V(r)}{\partial r}Y_{\lambda K}(\theta\phi)\right|q_2\right\rangle, \tag{A9}$$

$$f^{L\pm1LK}(q_1q_2) = \left\langle q_1\left|\frac{\partial V(r)}{\partial r}\{\sigma Y_{L\pm1}(\theta\phi)\}_{LK}\right|q_2\right\rangle. \tag{A10}$$

APPENDIX 2

$$b_{g_1g_2g_3}=\frac{1}{6}\frac{(1+\delta_{g_1g_2}+\delta_{g_2g_3}+\delta_{g_1g_3}+2\delta_{g_1g_2}\delta_{g_1g_3})^{1/2}}{[1+\frac{1}{2}\delta_{K_0 0}(3-\delta_{\mu_1 0}-\delta_{\mu_2 0}-\delta_{\mu_3 0}]^{1/2}}, \tag{A11}$$

$$\mathscr{K}^{K^0}(g_2\lambda_1\mu_1 i_1'|\lambda_1\mu_1 i_1 g_2)$$
$$=-\delta_{\mu_1+\mu_2,K_0}\frac{1}{1+\delta_{g_1g_2}}$$
$$\times\sum_{q_1q_2q_3q_4}\delta_{K_2K_3}\Big[\psi^{\lambda_1\mu_1 i_1'}_{q_1q_3}\psi^{\lambda_1\mu_1 i_1}_{q_1q_2}\psi^{g_2}_{q_4q_2}\psi^{g_2}_{q_4q_3}$$
$$\times-\phi^{\lambda_1\mu_1 i_1'}_{q_1q_3}\phi^{\lambda_1\mu_1 i_1}_{q_1q_2}\phi^{g_2}_{q_4q_2}\phi^{g_2}_{q_4q_3}\Big]\Big[\delta_{K_1-K_2,\mu_1}\delta_{K_4-K_2,\mu_2}$$
$$\times+\delta_{K_2-K_1,\mu_1}\delta_{K_2-K_4,\mu_2}+\delta_{K_2-K_1,\mu_1}\delta_{K_2+K_4,\mu_2}$$
$$+\delta_{K_1+K_2,\mu_1}\delta_{K_2-K_4,\mu_2}+\delta_{K_1+K_2,\mu_1}\delta_{K_2+K_4,\mu_2}\Big], \tag{A12}$$

$$\mathscr{K}^{K^0}(g_2\lambda_1\mu_1 i_1'|\lambda_1\mu_1 i_1 g_2)$$
$$=-\delta_{\mu_1-\mu_2,K_0}\frac{1}{1+\delta_{g_1g_2}}$$
$$\times\sum_{q_1q_2q_3q_4}\delta_{K_2K_3}\Big[\psi^{\lambda_1\mu_1 i_1'}_{q_1q_3}\psi^{\lambda_1\mu_1 i_1}_{q_1q_2}\psi^{g_2}_{q_4q_2}\psi^{g_2}_{q_4q_3}$$
$$\times-\phi^{\lambda_1\mu_1 i_1'}_{q_1q_3}\phi^{\lambda_1\mu_1 i_1}_{q_1q_2}\phi^{g_2}_{q_4q_2}\phi^{g_2}_{q_4q_3}\Big]\Big[\delta_{K_1-K_2,\mu_1}\delta_{K_2-K_4,\mu_2}$$
$$+\delta_{K_2-K_1,\mu_1}\delta_{K_4-K_2,\mu_2}$$
$$+\delta_{K_1-K_2,\mu_1}\delta_{K_2+K_4,\mu_2}+\delta_{K_1+K_2,\mu_1}\delta_{K_4-K_2,\mu_2}\Big], \tag{A13}$$

$$U^{g_0}_{g_1g_2}=\frac{1}{2}\sum_{n=1}^{n_{\max}}\sum_{\tau}\sum_{qq'}{}^{\tau}\Big\{V^{g_1}_{n\tau}(qq')T^{g_0,g_2}_{qq';\mu_1}$$
$$+V^{g_2}_{n\tau}(qq')T^{g_0,g_1}_{qq';\mu_2}\Big\}, \tag{A14}$$

$$T^{g_2,g_0}_{qq';\mu_1}=\sum_{q_3}{}^{\tau}(\psi^{g_0}_{q_3q'}\psi^{g_2}_{q_3q}+\phi^{g_0}_{q_3q'}\phi^{g_2}_{q_3q})$$
$$\times\theta^{\mu_1\mu_2K_0}_{KK'K_3}(1+\delta_{K_0 0})(1+\delta_{K_0 0}(1-\delta_{\mu_1 0})),$$

where $\theta^{\mu_1\mu_2K_0}_{KK'K_3} = -1$, if $K+K'=\mu$ or $K_3+K'=\mu$ or $\mu_1+\mu_2=K_0$ or $\mu_1-\mu_2=K_0$ and is equal to 1 otherwise.

[1] V. G. Soloviev, *Theory of Complex Nuclei* (Pergamon Press, Oxford, 1976) [Russ. original, Nauka, Moscow, 1971].
[2] V. G. Soloviev, Prog. Part. Nucl. Phys. **19**, 107 (1987).
[3] V. G. Soloviev, *Nuclear Theory. Quasiparticles and Phonons* (Énergoizdat, Moscow, 1989) [English transl., Institute of Physics Publishing, Bristol, 1992].
[4] V. G. Soloviev, Z. Phys. A **338**, 271 (1991).
[5] V. G. Soloviev, Prog. Part. Nucl. Phys. **28** (1992).
[6] Y. Yamaguchi, Phys. Rev. **95**, 1628 (1954).
[7] V. G. Soloviev and N. Yu. Shirikova, Yad. Fiz. **55**, 2359 (1992) [Sov. J. Nucl. Phys. **55**, 1310 (1992)]; Nucl. Phys. **A542**, 410 (1992).
[8] V. G. Soloviev, Z. Phys. **334**, 143 (1989).
[9] D. Karadzhov, V. G. Soloviev, and A. V. Sushkov, Izv. Akad. Nauk SSSR, Ser Fiz. **53**, 2150 (1989).
[10] I. Ragnarsson and R. A. Broglia, Nucl. Phys. **A263**, 263, 315 (1976).
[11] V. G. Soloviev, A. V. Sushkov, and N. Yu. Shirikova, Yad. Fiz. **53**, 101 (1991); **54**, 1232 (1991) [Sov. J. Nucl. Phys. **53**, 65 (1991); **54**, 748 (1991)].
[12] V. G. Soloviev and A. V. Sushkov, Z. Phys. A **345**, 155 (1993).
[13] V. G. Soloviev, A. V. Sushkov, and N. Yu. Shirikova, J. Phys. G **20**, 113 (1994).
[14] V. G. Soloviev, A. V. Sushkov, and N. Yu. Shirikova, Nucl. Phys. A (1994) (to be published).
[15] V. G. Soloviev and A. V. Sushkov, Preprint INS-Rep. 927, University of Tokyo (1992).
[16] V. G. Soloviev and N. Yu. Shirikova, Z. Phys. A **334**, 149 (1989).
[17] V. G. Soloviev and N. Yu. Shirikova, Izv. Akad. Nauk SSSR, Ser. Fiz. **54**, 818 (1990).
[18] T. Suzuki and H. Sagawa, Prog. Theor. Phys. **65**, 565 (1981).
[19] V. G. Soloviev and A. V. Sushkov, Phys. Lett. **262B**, 189 (1991).
[20] R. Piepenbring and M. K. Jammari, Nucl. Phys. **A481**, 81 (1988).
[21] V. O. Nesterenko, V. G. Soloviev, and A. V. Khalkin, Yad. Fiz. **32**, 1209 (1980) [Sov. J. Nucl. Phys. **32**, 625 (1980)].
[22] D. Karadjov, V. V. Voronov, and F. Catara, Phys. Lett. **306B**, 197 (1993).
[23] V. G. Soloviev and L. A. Malov, Nucl. Phys. **A196**, 433 (1972).
[24] G. Kurchev and V. G. Soloviev, Teor. Mat. Fiz. **22**, 244 (1975).
[25] M. G. Urin, *Relaxation of Nuclear Excitations* [in Russian] (Énergoatomizdat, Moscow, 1991).
[26] A. Bohr and B. R. Mottelson, *Nuclear Structure*, Vol. 1 (Benjamin, New York, 1969) [Russ. transl., Mir, Moscow, 1971].

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